۱۹٫۵

ساریناا اسپانی _ یازدهم ریاضی _ شماره تکلیف ۱۱

1)

$$f(x)=\begin{cases} x^3-4 & x>a \\ 12x-20 & x<a \end{cases}$$

$x^3-4=12x-20$

$x^3-12x+16=0$

$\to (x-2)^2(x+4)=0$

$$\begin{array}{r|l} \cancel{x^3}-12x+16 & x-2 \\ -\cancel{x^3}+2x^2 & x^2+2x-8 \\ \hline \cancel{2x^2}-12x+16 & \\ -\cancel{2x^2}+4x & \\ \hline -8x+16 & \\ +8x-16 & \\ \hline 0 & \end{array}$$

$$\begin{array}{c|c|c} & -4 & 2 \\ - & + & + \end{array}$$

$a\in[-4,+\infty)$

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2)

$f(x)=3x+k$ $\qquad f^{-1}(2)=4$ $\qquad f^{-1}(x)=\dfrac{x-k}{3}$

$f(4)=12+k=2 \to \boxed{k=-10} \to 3x-10$ $\qquad \dfrac{2-k}{3}=4 \to \boxed{k=-10}$

$f(7)=3(7)-10=11$

$f(f(x))=f(3x-10)=3(3x-10)-10=9x-30-10=9x-40$

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3)

$f(x)=\dfrac{ax}{x-1}$ $\qquad A(2a,a)$

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$f^{-1}(x) \Rightarrow x=\dfrac{ay}{y-1} \to xy-x=ay \to y=\dfrac{x}{x-a}$

$f^{-1}(x) \to \dfrac{x}{x-a} \xrightarrow{A(2a,a)} \dfrac{2a}{2a-a}=a=2$

$f=\{(۳,۵),(۴,۷),(۷,۲),(۹,۶)\}$ $g=\{(۳,۲),(۸,۶),(۹,۵)\}$ ④

الف) $f\circ f^{-1}=\{(۵,۵),(۷,۷),(۲,۲),(۶,۶)\}$

ب) $f^{-1}\circ f=\{(۳,۳),(۴,۴),(۷,۷),(۹,۹)\}$ ۳

ج) $f\circ g^{-1}=\{(۲,۵),(۵,۶)\}$

د) $g^{-1}\circ f=\{(۳,۹),(۹,۸),(۷,۳)\}$

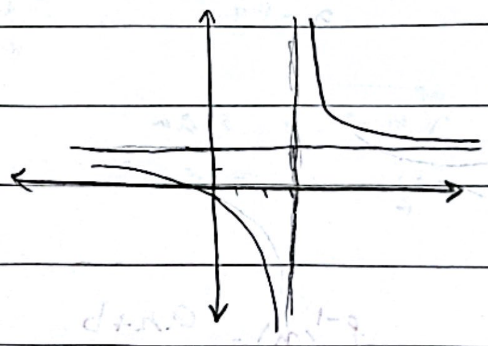
$\dfrac{h}{f\circ g^{-1}}=\searrow$ $g^{-1}(x)=\{(۱,۲),(۳,۴),(۵,۶)\}$ ⑤

$\dfrac{h}{f\circ g^{-1}}=\{(۱,\frac{۱}{۲}),(۳,\frac{۱}{۲})\}$ ۳

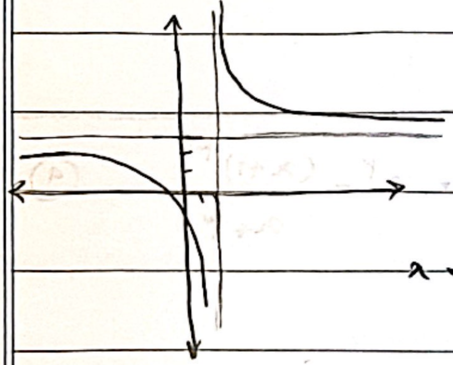
$\to ۱\to ۲\to ۴$ $۳\to ۴\to ۸$ $۵\to ۶\to ۰$

$f\circ g^{-1}=\{(۱,۴),(۳,۸),(۵,۰)\}$

$y=\dfrac{۳x+۱}{x-۲}$ $y=\dfrac{۲x+۱}{x-۳}$ ⑥



$y=\dfrac{۳x+۱}{x-۲}$



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$x=\dfrac{۳y+۱}{y-۲}\to y=\dfrac{۲x+۱}{x-۳}$

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$f(x)=|x-1|-|x-3|$

	1		3
-2		$2x-4$	2

$\rightarrow a=1$, $b=3$

$f^{-1}(x)=\frac{x+4}{2}$

$-2\le x\le 2$

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$x=2y-4 \rightarrow \frac{x+4}{2}=y$

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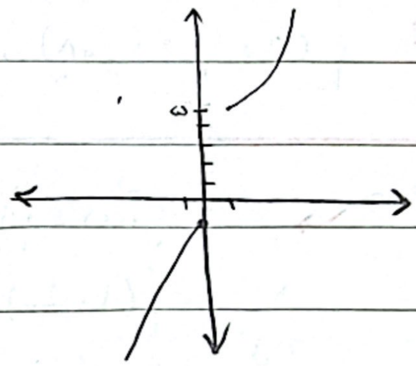
$$f(x)=\begin{cases} x^3+4 & x\geq 1 \\ 4x-1 & x\leq 0 \end{cases}$$



x	0	-1
y	-1	-5

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$x=y^3+4 \rightarrow \sqrt[3]{x-4}=y$

$D_{f^{-1}}=R_f \rightarrow 1 \rightarrow 1^3+4=5$

$D_{f^{-1}}=[5,+\infty)$

$x=4y-1 \rightarrow \frac{x+1}{4}=y \rightarrow D_{f^{-1}}=(-\infty,-1]$

$$f^{-1}(x)=\begin{cases} \sqrt[3]{x-4}\ , & x\geq 5 \\ \frac{x+1}{4}\ , & x\leq -1 \end{cases}$$

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$f^{-1}(b)=?$ $\qquad f^{-1}(x)=\frac{ax+b}{2x+d}$ $\qquad f(x)=x^2-\frac{(x+1)^3}{x+3}$

$$f(x)=\frac{x^3+3x^2-x^3-1-3x^2-3x}{x+3}=\frac{-(1+3x)}{x+3}$$

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$$x=\frac{-(1+3y)}{y+3} \rightarrow xy+3x=-1-3y$$

$$y=\frac{-(1+3x)}{x+3}=f^{-1}(x)$$

$$\frac{(-1-3x)\times 2}{(x+3)\times 2} = \frac{ax+b}{2x+d} \longrightarrow \frac{-6x-2}{2x+6} = \frac{ax+b}{2x+d} \longrightarrow \boxed{b=-2}$$

$$f^{-1}(-2) = \frac{-1+6}{1} = 5$$

$$f(x) = \frac{x}{x^2+1} \longrightarrow x = \frac{y}{y^2+1} \longrightarrow xy^2 + x = y$$ 10

$$xy^2 - y + x = 0$$ 5

$$\longrightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \xrightarrow[\text{هم علامت‌اند}]{} y \text{ و } x$$

$$y = \frac{1+\sqrt{1-4x^2}}{2x}$$