نام و نام خانوادگی .....دیا.....پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱۸,۲۵

۱

$f=\{(3,2)(n,a)(3,n^2-n)(m,2)(-1,4)\}$

$m=2$

$n^2-n=2 \rightarrow n^2-n-2=0$

$(n-2)(n+1)=0$ → $n=2$ یا $n=-1$

$f=\{(-1,1)(1,2)(2,3)(a,m-1)(a+2,n)(m,4)\}$

$(a,1)$

$m=2$ و $n=2$

$a=-1$

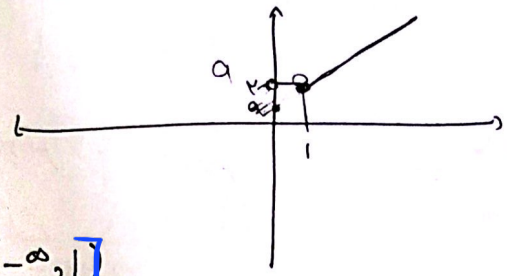
۲

$f(x)=\begin{cases} 3x-1 & x\geq 1 \\ x+a & x<1 \end{cases}$

$x+a \xrightarrow{x=1} 1+a=2$

$a=1$

→ $(-\infty,1]$



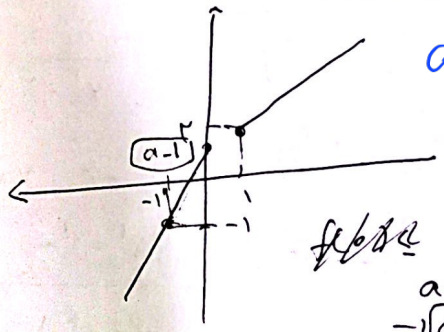
۱,۷۵

۳

$f(x)=\begin{cases} 3x-1 & x\geq 1 \\ ax+a-1 & x\leq 0 \end{cases}$

$0\leq a-1\leq 2$

$\rightarrow a\in[1,3]$



$a>0$

$0<a<3$

$(-\infty,3)$

$a-1<2$

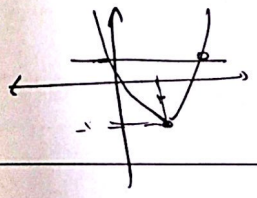
$\rightarrow a<3$

۱,۵

۴

$y=x^3+2 \rightarrow \sqrt[3]{y-2}=x \rightarrow \sqrt[3]{x-2}=y$

$y=x^2-4x+2 \rightarrow \begin{cases} \frac{-b}{2a}=\frac{4}{2}=2 \\ \frac{-\Delta}{4a}=\frac{-(16-8)}{4}=-2 \end{cases}$



تابع وارون پذیر است

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۵

$y=\frac{3x+1}{x-2}$ ✓ → $xy-2x=3y+1$

→ $x=\frac{3y+1}{y-2}$ → $y(x-3)=1+2x$

→ $y=\frac{1+2x}{x-3}$

$y=\frac{4+2x}{x+2}$ → تابع ثابت است

۲

$f(x+\sqrt{x}) = x$

$x = y + \sqrt{y} \rightarrow x^2 = y^2 + y + 2y\sqrt{y}$

$y = \sqrt{x-3} \rightarrow y^2 = x-3 \rightarrow y^2+3 = x$

$\rightarrow x^2+3 = y$

$y = x^2 - 4x + 3,\ x \in [2,+\infty) \rightarrow$ وارون پذیر

$y = (x-2)^2 - 1$

$\sqrt{y+1} = x-2 \rightarrow \sqrt{y+1}+2 = x$

$y = \sqrt{x+1} + 2$

$f^{-1}(x) \rightarrow ax+b-\sqrt{x-c}$

$D_f = [0,+\infty)$

$f(x) = x+\sqrt{x}$

$f(1) = 2 \rightarrow f^{-1}(2) = 1$

$f(3) = 4 \rightarrow f^{-1}(4) = 3$

$f^{-1}(f(x+\sqrt{x})) = x \rightarrow ax+b-\sqrt{x-c}$

$f^{-1}(2) = 1$

$a+2b+4c = 1$

$a+4b+4c = 4$

$a = 1$

$b = \frac{1}{2}$

$b \neq 0$

$x = c$

$4 - \sqrt{2-c} = 1 \rightarrow \sqrt{2-c} = 3$

$2-c = 9 \rightarrow c = -7$

$c = -\frac{1}{2}$

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow \frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \rightarrow \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2}$

$\rightarrow x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2 \rightarrow x_1 = \pm x_2 \rightarrow x_1 = x_2$

$y = \frac{x\sqrt{2}}{\sqrt{x^2-1}}$

$x = \frac{y}{\sqrt{y^2-2}} \rightarrow x^2 = \frac{y^2}{y^2-2} \Rightarrow y^2x^2 - 2x^2 = y^2 \rightarrow y^2 = \frac{2x^2}{x^2-1} \rightarrow y = \sqrt{\frac{2x^2}{x^2-1}}$

$f^{-1}(x) = \frac{x}{1+|x|}$

$f(\frac{x}{1+|x|}) = x$

$\frac{x}{1+|x|} = -\frac{2}{3} \rightarrow 3x = -2-2|x| \rightarrow 5x = -2-2|x|$

$x < 0 \rightarrow 5x = -2+2x \rightarrow 3x = -2 \rightarrow x = -\frac{2}{3}$

$f(-\frac{2}{3}) + g^{-1}(-\frac{2}{3}) = -\frac{28}{75}$

$g(-\frac{2}{3}) = 5 \rightarrow g(5) = -\frac{2}{3}$

$g(x) = \sqrt{x} - 1 \rightarrow \sqrt{x} = -\frac{2}{3} + 1 \rightarrow \sqrt{x} = \frac{1}{3} \rightarrow x = \frac{1}{9}$

$f^{-1}(x) = \sqrt[3]{x-1}$

$f(\sqrt[3]{x-1}) = x \rightarrow$ جایگذاری

$g^{-1}(12) = ?$

$g(x) = f(x) + \sqrt{f(x)} = 12 \rightarrow f(x) = 9$

$f^{-1}(x) = x$