پارمیس عباسیان تمارین تکلیف = ۱۰

۱) الف) $f=\{(3,2),(x,5),(3,x^2-x),(4,2),(-1,4)\}$

$x^2-x=2 \to x^2-x-2=0 \to (x-2)(x+1)$ $\begin{cases} x=-1 \ \times \\ x=2 \ \checkmark \end{cases}$

ب) $f=\{(-1,1),(1,2),(2,3),(a,m-1),(a+1,8),(m,3)\}$

$m=2$

$a=-1 \to a+2=1 \to n=2$

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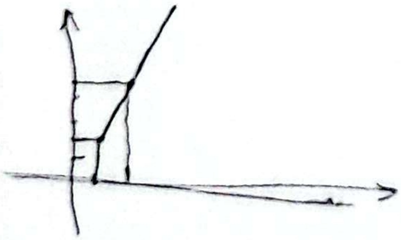
۲) $f(x)=\begin{cases} 3x-1 & x\geq 1 \\ x+a & x<1 \end{cases} \to (-\infty,2) \to x+a<2 \to a>1$



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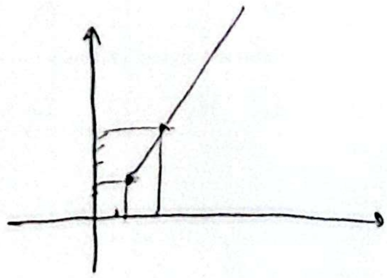
۳)

$f(x)=\begin{cases} 3x-1 & x\geq 1 \\ ax+a-1 & x<0 \end{cases} \to (-\infty,2) \to ax+a-1<2$

$a-1<2 \to a<3$



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۴) الف) $y=x^3+2$

$\begin{matrix} y_1=x_1^3+2 \\ y_2=x_2^3+2 \end{matrix} \to x_2^3+\cancel{2}=x_1^3+\cancel{2}$

$x_2^3=x_1^3 \to x_1=x_2$

$x=y^3+2 \to y^3=x-2 \to y=\sqrt[3]{x-2}$

ب) $y=x^2-4x+2$

$\left.\begin{matrix} y=(x_1-2)^2-2 \\ y=(x_2-2)^2-2 \end{matrix}\right\} \to (x_1-2)^2-\cancel{2}=(x_2-2)^2-\cancel{2}$

$x_1=\pm x_2$ ✗

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۵) الف) $y=\dfrac{3x+1}{x-2}$ وارون $x=-\dfrac{-2y-1}{y-3}=y=\dfrac{2x+1}{x-3}$

ب) $y=\dfrac{4+2x}{x+2}$ $\quad y=\dfrac{2\cancel{(x+2)}}{\cancel{x+2}}=2$ ✗

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الف) $y=\sqrt{x-3}$

$y=\sqrt{x_1-3}$, $y=\sqrt{x_2-3}$ $\Big\} \to \sqrt{x_1-3}=\sqrt{x_2-3}$

$x_1-\cancel{3}=x_2-\cancel{3}$ ✓

$x=\sqrt{y-3} \Rightarrow x^2=y-3 \to y=x^2+3$

ب) $y=x^2-4x+3$ $\begin{cases} -\frac{b}{2a}=2 \\ -1 \end{cases} \Rightarrow [2,+\infty)$

$y=(x-2)^2-1$

$x(y-2)^2-1 \to x-1=(y-2)^2$

$\sqrt{x-1}=y-2 \to y=\sqrt{x-1}+2$

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7) $f(x)=x+\sqrt{x} \to R=[0,+\infty)$ $\Rightarrow f^{-1}(x)=ax+b-\sqrt{x-c} \to R_{f^{-1}}=[0,+\infty)$

$f^{-1}(x)=x=y+\sqrt{y} \to x=y+\sqrt{y}+\frac{1}{4}-\frac{1}{4} \to x+\frac{1}{4}=(\sqrt{y}+\frac{1}{2})^2,\ \sqrt{x+\frac{1}{4}}=\sqrt{y}+\frac{1}{2}$

$\sqrt{y}=-\frac{1}{2}+\sqrt{x+\frac{1}{4}} \to y=\frac{1}{4}+x+\frac{1}{4}-\sqrt{x+\frac{1}{4}} \to y=x-\sqrt{x+\frac{1}{4}}+\frac{1}{2}$

$\underset{1}{a}x+\underset{\frac{1}{2}}{b}-\sqrt{x-\underset{-\frac{1}{4}}{c}}$

$a+2b+4c=1+1-1=1$

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8)

$y=\frac{x}{\sqrt{x^2-2}} \to y_1=y_2 \to \frac{x_1}{\sqrt{x_1^2-2}}=\frac{x_2}{\sqrt{x_2^2-2}} \to \frac{x_1^2}{x_1^2-2}=\frac{x_2^2}{x_2^2-2} \to \frac{x_1^2+2-2}{x_1^2-2}$

$1+\frac{2}{x_1^2+2}=1+\frac{2}{x_2^2-2} \to \frac{2}{x_1^2-2}=\frac{2}{x_2^2-2}=x_1^2\cancel{-2}=x_2^2\cancel{-2}$ $x_1=\pm x_2$

امّا یک به یک

$x=\frac{y}{\sqrt{y^2-2}}=x^2=\frac{y^2}{y^2-2} \to x^2y^2-2x^2=y^2 \to y^2(x^2-1)=2x^2 \Rightarrow y=\frac{\sqrt{2}x}{\sqrt{x^2-1}}$

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9) $f^{-1}(x)=\frac{x}{1+|x|} \to g(x)=\sqrt{x}-1$ $f(x)=\begin{cases} x\geq 0 & \frac{x}{1-x} \\ x<0 & \frac{x}{1+x} \end{cases}$ $f(-\frac{2}{5})=-\frac{2}{3}$

$g^{-1}(x)=y=x+1 \to g^{-1}(x)=(x+1)^2 \to g^{-1}(-\frac{2}{5})=\frac{9}{5}$

$f(-\frac{2}{5})+g^{-1}(-\frac{2}{5})=-\frac{2}{3}+\frac{9}{25}=-\frac{23}{75}$

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10. $f^{-1}(x)=\sqrt[3]{x-1} \to x=\sqrt[3]{y-1} \to y-1=x^3 \Rightarrow y=x^3+1$

$g(x)=f(x)+\sqrt{f(x)} \xrightarrow{x\geq -1} g(x)=\underbrace{x^3+1}_{b^2}+\underbrace{\sqrt{x^3+1}}_{b}=12 \to b^2-b-12$ $\begin{cases} b=-4 \\ b=3 ✓ \end{cases}$

$\sqrt{x^3+1}=3 \to x^3=8$ $x=2$

$g^{-1}(12)=b$ $g(b)=12 \to g^{-1}(12)=2$

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