

1

$$\frac{x_1}{\sqrt{x_1^2 - 1}} = y \quad \frac{x_2}{\sqrt{x_2^2 - 1}} = y \quad \frac{x_1^2}{x_1^2 - 1} = \frac{x_2^2}{x_2^2 - 1} \quad x_1^2 x_2^2 - 1 x_1^2 = x_2^2 x_1^2 - 1 x_2^2$$

$$x_1^2 = x_2^2 \xrightarrow{\text{take sq. rt.}} \sqrt{x_1^2} = \sqrt{x_2^2}$$

$$f^{-1}(x) \Rightarrow x = \frac{y}{\sqrt{y^2 - 1}} \quad x \sqrt{y^2 - 1} = y \quad x^2 (y^2 - 1) = y^2$$

$$y = \frac{x}{\sqrt{x^2 - 1}} \xrightarrow{\text{cross multiply}} y \sqrt{x^2 - 1} = x$$

2

$$y = \frac{x}{1+x} \quad y + yx = x \quad y = \frac{x}{1+x} \quad x$$

$$y = \frac{x}{1+x} \quad y - yx = x \quad y = \frac{x}{1+x} \quad (-\frac{1}{x}) \checkmark$$

$$f(x) = \sqrt{x} - 1 \quad \sqrt{x} = y + 1 \quad y = (x+1)^{-\frac{1}{2}}$$

$$f(-\frac{1}{3}) + f^{-1}(-\frac{1}{3}) = -\frac{1}{\sqrt{3}}$$

3

$$y = x^{\frac{1}{2}+1} \quad g(x) = x^{\frac{1}{2}+1} + \frac{\sqrt{x^{\frac{1}{2}+1}}}{+}$$

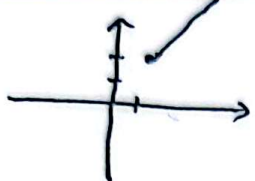
$$y = t^{\frac{1}{2}+1} \quad t^{\frac{1}{2}+1} - y = 0 \quad t = \frac{-1 + \sqrt{1+4y}}{2} \quad t = x^{\frac{1}{2}+1}$$

$$x = \sqrt[3]{t^{\frac{1}{2}+1}} \quad x = \sqrt[3]{\left(\frac{-1 + \sqrt{1+4y}}{2}\right)^{\frac{1}{2}+1}} \quad \frac{g^{-1}(12)}{\sqrt{12}} = 12$$

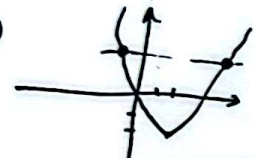
1) $n^2 - n - 2 \rightarrow n^2 - n - 2 = 0 \quad (n-2)(n+1) = 0 \rightarrow n = 2 \quad \text{و} \quad n = -1 \rightarrow m = 2$

$\Rightarrow m = 2 \rightarrow a = 1 \rightarrow n = 2$

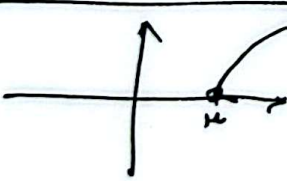
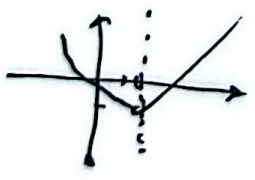
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2)  $x + a \leq 1 \rightarrow 1 + a \leq 1 \rightarrow a \leq 0 \quad (-\infty, 0]$

3) $ax + a - 1$  $\begin{cases} a > 0 \rightarrow a - 1 < 1 \rightarrow a < 2 \\ a < 0 \rightarrow a - 1 > 1 \rightarrow a > 2 \end{cases} \rightarrow a < 2 \text{ و } a > 2$

4) $\begin{cases} y = x^2 + 1 \\ y = x^2 + 2 \end{cases} \rightarrow x_1^2 + 1 = x_2^2 + 2 \rightarrow x_1^2 - x_2^2 = 1 \rightarrow x_1 = x_2 \quad \checkmark$
 $\rightarrow x = y^2 + 1 \rightarrow y^2 = x - 1 \rightarrow y = \sqrt{x-1}$
 5)  x y

5) $\text{الف) } \sqrt{x^2 + 1} = x + 1 \rightarrow x = \frac{y+1}{y-1} \quad y(x-1) = 1+y \rightarrow y = \frac{1+y}{x-1}$
 $\Rightarrow y = \frac{1+y}{x-1} \quad \frac{y(1+y)}{x-1} = 1 \quad x = 1 \text{ و } x = 2$

6)  $\sqrt{x^2 + 1} \rightarrow F^{-1}(x) \Rightarrow x = \sqrt{y-1} \rightarrow x^2 = y-1 \rightarrow y = x^2 + 1$
 $\Rightarrow$  $x = y^2 - 1 \rightarrow x = (y-1)^2 - 1$
 $y = 1 \pm \sqrt{x+1}$
 $\xrightarrow{x=y} F^{-1}(x) = 1 + \sqrt{x+1}$

7) $\sqrt{x+1} \rightarrow y = x^2 + 1 \quad x^2 + 1 - y = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+4y}}{2}$
 $t = \sqrt{x} \rightarrow y = \left(\frac{-1 \pm \sqrt{1+4y}}{2} \right)^2 \rightarrow \frac{1 \pm 2\sqrt{1+4y} + 1}{4} = \frac{1 \pm 2\sqrt{1+4y}}{4}$
 $x + \frac{1}{x} = \frac{1}{x} \sqrt{1+4x} \rightarrow x + \frac{1}{x} = \sqrt{x+1} \rightarrow a = 1 \quad b = \frac{1}{x} \quad c = -\frac{1}{x}$
 $a + 1b + 1c = 1$