یازدهم ریاضی C — $18,75$ — تکلیف ۶ - ۱۰

Date No

1- الف) $f=\{(3,2)(n,5)(3,n^2-n)(m,2)(-1,4)\}$

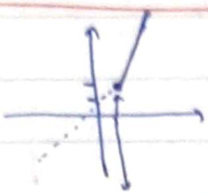
$\to n^2-n=2 \Rightarrow n^2-n-2=0 \to (n-2)(n+1)=0 \to \begin{cases} n=-1 \to \{f\ldots(-1,5)(-1,4)\} \to \times \text{ تابع نیست} \\ n=2\ \checkmark \end{cases}$

$\to m=3$

ب) $f=\{(-1,1)(1,2)(2,3)(a,m-1)(a+2,n)(m,3)\}$

$\to m=2 \Rightarrow m-1=1 \Rightarrow a=-1 \Rightarrow a+2=1 \Rightarrow n=2$

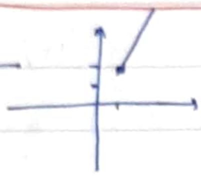
2- $f(x)=\begin{cases} 2x-1\ ; & x\geq 1 \\ x+a\ ; & x<1 \end{cases}$ —



$\Rightarrow max \to \left|\begin{smallmatrix}1\\1\end{smallmatrix}\right. \Rightarrow a=(-\infty,1]$

$\to a\notin(1,+\infty)$

3- $f(x)=\begin{cases} 3x-1\ ; & x\geq 1 \\ ax+a-1\ ; & x\leq 0 \end{cases}$ —



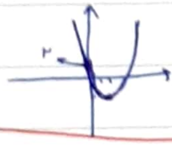
① $max \to \left|\begin{smallmatrix}0\\2\end{smallmatrix}\right. \Rightarrow a-1=2 \Rightarrow a=3 \Rightarrow a\leq 3 \xrightarrow{a\neq 3} a<3$

② $ax+(a-1) \to a>0 \overset{\cap}{\Rightarrow} a=(0,3)$

4- الف) $y=x^3+2$ —

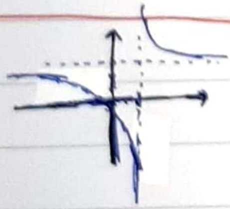
$\Rightarrow$ یک به یک ✓ $\to x^3=y-2 \to x=\sqrt[3]{y-2} \to y=\sqrt[3]{x-2} \to f^{-1}(x)=\sqrt[3]{x-2}$

ب) $y=x^2-4x+2 \to ext\left|\begin{smallmatrix}2\\-2\end{smallmatrix}\right.$ —



$\to$ یک به یک ×

5- الف) $y=\frac{3x+1}{x-2}$ —



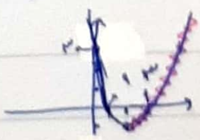
$\to$ یک به یک ✓ $\to x=\frac{3y+1}{y-2} \to y=\frac{2x+1}{x-3} \to f^{-1}(x)=\frac{2x+1}{x-3}$

ب) $y=\frac{4+2x}{x+2}=2 \to \times$ یک به یک

6- الف) $y=\sqrt{x-3} \to y_1=y_2 \to \sqrt{x_1-3}=\sqrt{x_2-3} \xrightarrow{()^2} x_1-3=x_2-3 \Rightarrow x_1=x_2 \Rightarrow$ یک به یک

$\to x=\sqrt{y-3} \to x^2=y-3 \to y=x^2+3 \to f^{-1}(x)=x^2+3$

ب) $y=x^2-4x+3,\ x\in[2,+\infty) \to ext\left|\begin{smallmatrix}2\\-1\end{smallmatrix}\right.$ —



$\leftarrow x\in[2,+\infty) \Rightarrow$ یک به یک

$\to x=y^2-4y+3 \xrightarrow{+1-1} x=(y-2)^2-1 \to y-2=\pm\sqrt{x+1} \Rightarrow y=2+\sqrt{x+1} \to f^{-1}(x)=2+\sqrt{x+1}$

$(R_{f^{-1}}\geq 2 \Rightarrow -$ غ ق ق$)$

7- $f(x)=x+\sqrt{x} \to y=x+\sqrt{x} \to x=y+\sqrt{y} \to x=\sqrt{y}^2+\sqrt{y}+\frac14-\frac14 \Rightarrow (\sqrt{y}+\frac12)^2-\frac14=x$

$\to \sqrt{y}+\frac12=\sqrt{x+\frac14} \to \sqrt{y}=\sqrt{x+\frac14}-\frac12 \Rightarrow y=x+\frac14+\frac14-\sqrt{x+\frac14} \to f^{-1}(x)=x+\frac12-\sqrt{x+\frac14}$

$f^{-1}(x)=ax+b-\sqrt{x-c}$

$\to a=1,\ b=\frac12,\ c=-\frac14 \Rightarrow a+2b+4c=1$

8- $y=\frac{x}{\sqrt{x^2-2}} \to \frac{x_1}{\sqrt{x_1^2-2}}=\frac{x_2}{\sqrt{x_2^2-2}} \to \frac{x_1^2}{x_1^2-2}=\frac{x_2^2}{x_2^2-2} \Rightarrow x_1^2x_2^2-2x_2^2=x_1^2x_2^2-2x_1^2 \Rightarrow x_1^2=x_2^2$

$\to x_1=\pm x_2 \Rightarrow \cancel{\text{یک به یک}}$

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9- $f^{-1}(x)=\frac{x}{1+|x|}$ $\quad g(x)=\sqrt{x}-1$ $\quad f(-\frac{2}{5})+g^{-1}(-\frac{2}{5})$.

$\frac{x}{1+|x|}=-\frac{2}{5} \to -5x=2+2|x| \to 5x+2|x|+2=0 \longrightarrow 5x-2x+2=0 \Rightarrow 3x+2=0 \Rightarrow x=-\frac{2}{3}$ ✓

$5x+2x+2=0 \Rightarrow 7x+2=0 \Rightarrow x=-\frac{2}{7} \to$ غ ق ق

$\Rightarrow f(-\frac{2}{5})=-\frac{2}{3}$ $\quad \sqrt{x}-1=-\frac{2}{5} \Rightarrow \sqrt{x}=\frac{3}{5} \Rightarrow x=\frac{9}{25} \Rightarrow g^{-1}(-\frac{2}{5})=\frac{9}{25}$

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$\Rightarrow f(-\frac{2}{5})+g^{-1}(-\frac{2}{5})=\frac{9}{25}-\frac{2}{3}=\frac{27-50}{75}=\boxed{-\frac{23}{75}}$

10- $f^{-1}(x)=\sqrt[3]{x-1}$ $\quad g(x)=f(x)+\sqrt{f(x)}$ $\quad g^{-1}(12)$

$\Rightarrow f(x)? \quad y=\sqrt[3]{x-1} \Rightarrow y^3=x-1 \Rightarrow x=y^3+1 \Rightarrow f(x)=x^3+1$

$\Rightarrow g(x)=x^3+1+\sqrt{x^3+1} \longrightarrow x^3+1+\sqrt{x^3+1}=12 \longrightarrow \sqrt{x^3+1}^{\,2}+\sqrt{x^3+1}-12=0$

$\sqrt{x^3+1}=3 \to x^3+1=9 \Rightarrow x^3=8 \Rightarrow x=2 \to \boxed{g^{-1}(12)=2}$

$\Rightarrow (\sqrt{x^3+1}-3)(\sqrt{x^3+1}+4)=0$

$\sqrt{x^3+1}=-4$ غ ق ق

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۸) یک y نمی‌تواند دو x با علامت مختلف داشته باشد
پس $x_1=x_2$ و تابع یک به یک است

$y=\frac{\sqrt{2x}}{\sqrt{x^2-1}}$

x و y هم علامت هستند پس نیازی به علامت ± نیست