

۱۹, ۲۵

الف) $(\mu, \nu), (n, \omega), (\mu, n^2 - n), (m, \nu), (-1, \varepsilon)$

1

$$m = \mu$$

$$n^2 - n = \nu \rightarrow n^2 - n - \nu = 0 \rightarrow (n - \nu)(n + 1) = 0 \rightarrow n = \nu, \quad \begin{matrix} \text{X} \rightarrow (-1, \varepsilon) \\ \checkmark \end{matrix}$$

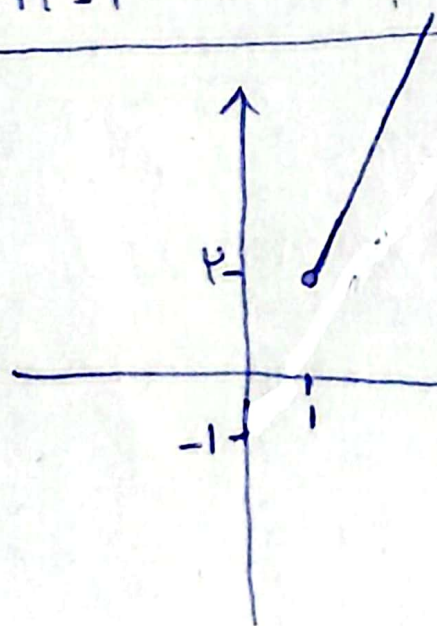
۲

ب) $(-1, 1), (1, \nu), (\mu, \mu), \underbrace{(a, m-1)}_{(-1, 1)}, \underbrace{(a+\nu, n)}_{(1, n)}, \underbrace{(m, \mu)}_{(\mu, \mu)}$

$$m = \nu$$

$$(a, 1) \rightarrow a = -1$$

$$n = \nu$$



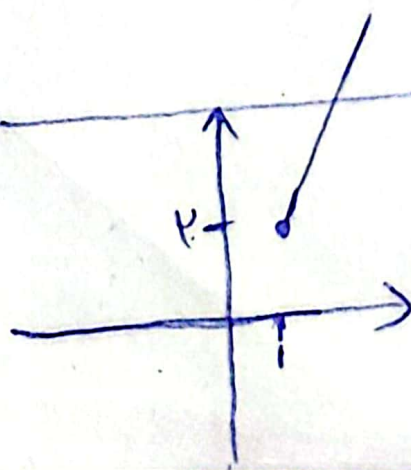
$$\mu = 1 = \nu \xleftarrow{x=1} \mu_{x-1} \quad \left. \begin{matrix} x > 1 \\ x < 1 \end{matrix} \right\} \quad \begin{matrix} x+a \\ x+a \end{matrix}$$

۱, ۷۵

$$x+a = \nu \rightarrow a = 1$$

$$\leftarrow x+a < \nu \quad \leftarrow \begin{matrix} x < 1 \\ y < \nu \end{matrix}$$

$$a < 1$$

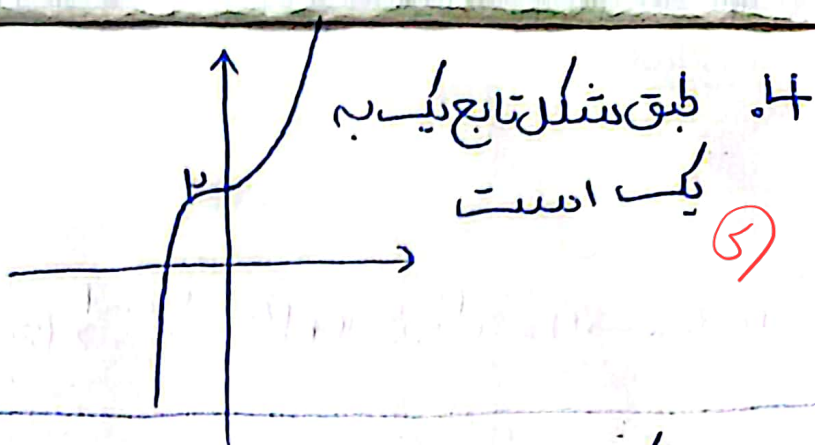


$$a x + a - 1 \quad \left. \begin{matrix} x < 0 \end{matrix} \right\} \quad \begin{matrix} x = 0 \rightarrow a - 1 < \nu \rightarrow a < \mu \\ a < 0 \end{matrix}$$

۱, ۷۵

الف) $y = x^3 + 2$

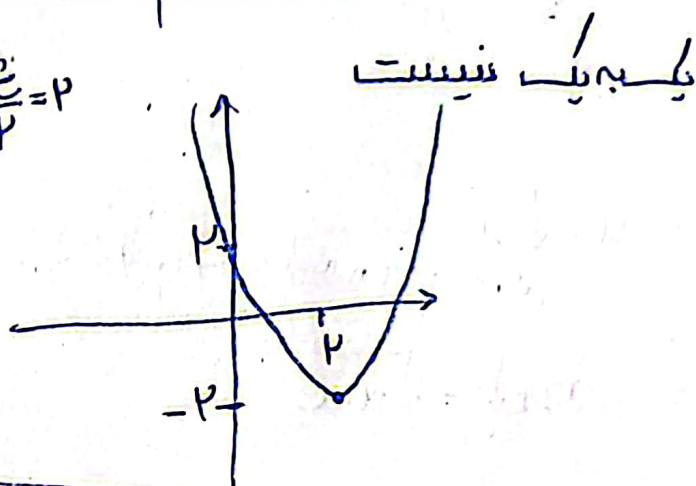
معکوسه $y = \sqrt[3]{x-2}$



ب) $y = x^2 - \varepsilon x + 2$

$y = \varepsilon - 1 + 2 = -2$

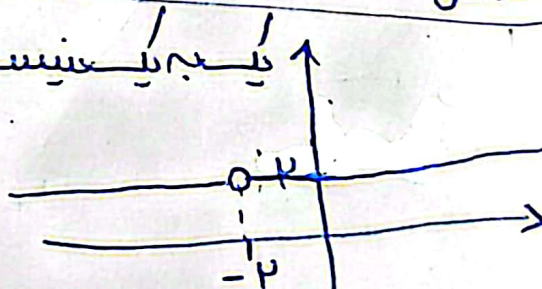
$\frac{-b}{2a} = \frac{\varepsilon}{2} = 2$



الف) $y = \frac{3x+1}{x-2}$ چون همواره افقی است پس یک به یک است.

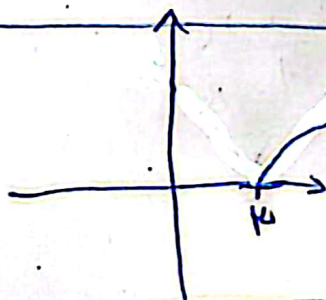
$yx - 2y = 3x + 1 \rightarrow 3x - yx = 1 + 2y \rightarrow x(3-y) = 1 + 2y \rightarrow x = \frac{1+2y}{3-y}$

ب) $y = \frac{\varepsilon + 3x}{x+2} = \frac{2(x+2)}{x+2} \rightarrow y = 2$ یک به یک نیست



الف) $y = \sqrt{x-3}$

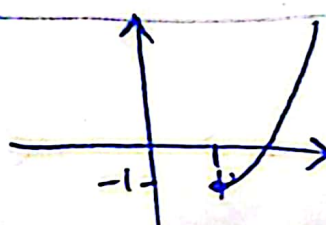
معکوسه $y = x^2 + 3$



6. یک به یک هست

ب) $y = x^2 - \varepsilon x + 3$ $x \in [2, +\infty)$

$\frac{-b}{2a} = \frac{\varepsilon}{2} = 2 \rightarrow \varepsilon - 1 + 3 = -1$



یک به یک هست

$$\text{Solve } \rightarrow y = x^p - \varepsilon x + \mu \xrightarrow{+1-1} (x-p)^p - 1 = y \rightarrow$$

$$y = \sqrt{x+1} + p$$

$$f(x) = x + \sqrt{x} \quad (0, +\infty)$$

7

$$f(x) = ax + b - \sqrt{x - c}$$

5

$$x + \sqrt{x} + \frac{1}{\varepsilon} - \frac{1}{\varepsilon} \rightarrow (\sqrt{x} + \frac{1}{p})^p - \frac{1}{\varepsilon} = y \rightarrow \sqrt{x} + \frac{1}{p} = \sqrt{y + \frac{1}{\varepsilon}} \rightarrow$$

$$x = \left(\sqrt{y + \frac{1}{\varepsilon}} - \frac{1}{p} \right)^p \rightarrow y = \left(\sqrt{x + \frac{1}{\varepsilon}} - \frac{1}{p} \right)^p \rightarrow y^p = x + \frac{1}{\varepsilon} + \frac{1}{\varepsilon} - \sqrt{x + \frac{1}{\varepsilon}}$$

$$1 + 1 - 1 = 1$$

$$a = 1$$

$$b = \frac{1}{p} \quad c = -\frac{1}{\varepsilon}$$

$$\frac{x}{\sqrt{x^p - p}} = y \rightarrow y \sqrt{x^p - p} = x \rightarrow y^p (x^p - p) = x^p \rightarrow y^p x^p - p y^p = x^p \rightarrow x^p - y^p x^p = -p y^p \rightarrow x^p (-y^p + 1) = -p y^p \rightarrow x^p = \frac{-p y^p}{-y^p + 1} \rightarrow x = \sqrt{\frac{p y^p}{y^p - 1}} \Rightarrow \frac{\sqrt{p y}}{\sqrt{y^p - 1}} = x \rightarrow y = \frac{\sqrt{p} x}{\sqrt{x^p - 1}} \leftarrow \text{مطلوب}$$

پایان

$$f^{-1}(x) = \frac{x}{1+|x|} \quad g(x) = \sqrt{x} - 1 \quad 9$$

$$\sqrt{x} - 1 = \frac{-p}{\omega} \rightarrow \sqrt{x} = \frac{\omega}{\omega} \rightarrow x = \frac{p}{\omega}$$

$$f\left(\frac{-p}{\omega}\right) + g^{-1}\left(\frac{-p}{\omega}\right) = ? \rightarrow \frac{-p}{\omega} + \frac{q}{p\omega} = \frac{-\omega + p}{p\omega} = \boxed{\frac{-p\omega}{p\omega}}$$

$$\frac{x}{1+|x|} = \frac{-p}{\omega} \rightarrow \omega x = -p - p|x| \quad \begin{matrix} x > 0 \\ x < 0 \end{matrix} \rightarrow \begin{matrix} \omega x = -p - p x \\ \omega x = -p + p x \end{matrix}$$

$$\omega x = -p + p x \rightarrow p x = -p \rightarrow x = \frac{-p}{p}$$

$$y = \sqrt{x-1} \rightarrow y^2 = x-1 \rightarrow x^2 + 1 = (y)$$

$$g(x) = x^2 + 1 + \sqrt{x^2 + 1} \rightarrow g^{-1}(11) = x^2 + 1 + \sqrt{x^2 + 1} \rightarrow x = 1$$