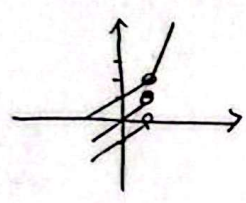


الف) $f_2 \{ (3,2), (n,5), (3, n^2-n), (m,2), (-1,4) \}$
 $\Rightarrow n^2-n \geq 2 \rightarrow n^2-n-4 \geq 0 \rightarrow \begin{cases} n \geq 1 \\ n \geq 3 \end{cases} \rightarrow \boxed{n \geq 3}$

ب) $f_2 \{ (-1,1), (1,2), (2,3), (a, m-1), (a+2, n), (m,3) \}$
 $\Rightarrow \boxed{m \geq 2} \rightarrow \boxed{a \geq -1} \rightarrow \boxed{n \geq 2}$

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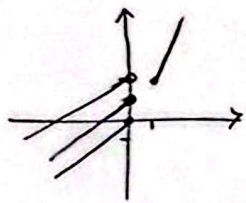
$f(n) = \begin{cases} 3n-1 & ; n \geq 1 \\ n+a & ; n < 1 \end{cases} \rightarrow f(1) = 2$
 $\rightarrow f(1) = a+1$



$\Rightarrow a+1 \leq 2 \rightarrow \boxed{a \leq 1}$

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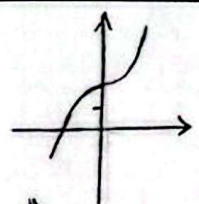
$f(n) = \begin{cases} 3n-1 & ; n \geq 1 \\ a+n+a-1 & ; n < 0 \end{cases} \rightarrow f(1) = 2$
 $\rightarrow f(0) = 2a-1$



$\Rightarrow \begin{cases} a \geq 0 \\ a+1 < 2 \rightarrow a < 1 \end{cases} \Rightarrow \boxed{0 \leq a < 1}$

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الف) $y = x^3 + 2$

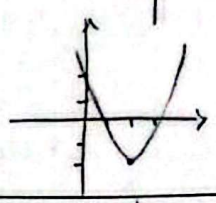


$\Rightarrow$ تابع یک به یک است $\Rightarrow$ معکوس $\Rightarrow \begin{cases} x = y^3 + 2 \\ \Rightarrow \boxed{y = \sqrt[3]{x-2}} \end{cases}$

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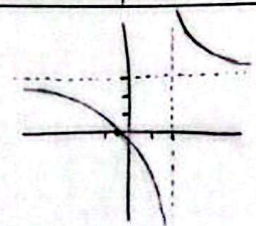
ب) $x^2 - 4x + 2$

$\left| \frac{-b}{2a} \right| = 2$
 $4 - 4 + 2 = 2$



$\Rightarrow$ تابع یک به یک نیست

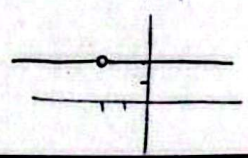
الف) $y = \frac{x+1}{x-2}$



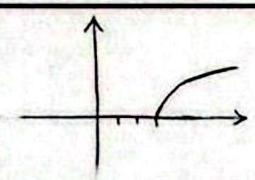
$\Rightarrow$ تابع یک به یک است $\Rightarrow$ معکوس $\Rightarrow \begin{cases} x = \frac{y+1}{y-2} \\ \Rightarrow \boxed{y = \frac{x+1}{x-2}} \end{cases}$

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ب) $y = \frac{x+2m}{m+2}$



$\Rightarrow$ تابع یک به یک نیست

الف) $y = \sqrt{n-3}$ 	$\Rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} n = \sqrt{y-3} \Rightarrow y = n^2+3$ $n \in [0, +\infty)$
ب) $y = n^2 - 4n + 3, n \in [2, +\infty)$ $\frac{b}{4a} = 1$ $f(-1+3) = -1$	$\Rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} n = \sqrt{y-1} + 2$ $\Rightarrow n = (y-1)^2 - 1 \Rightarrow y = \sqrt{n+1} + 2$
$f(n) = n + \sqrt{n} \xrightarrow{\text{عکس}} n = y + \sqrt{y} + \frac{1}{4} - \frac{1}{4} \rightarrow n = (y + \frac{1}{4})^2 - \frac{1}{4} \rightarrow y = (\sqrt{n + \frac{1}{4}} - \frac{1}{4})^2$ $\Rightarrow y = n + \frac{1}{4} - \sqrt{n + \frac{1}{4}} = an + b - \sqrt{n-c} \rightarrow$ $a = 1$ $b = \frac{1}{4}$ $c = -\frac{1}{4}$ $\Rightarrow a+b+c = 1 + \frac{1}{4} - \frac{1}{4} = 1$	
$y = \frac{n}{\sqrt{n^2-1}}$ $\xrightarrow{\text{تعریف}} f(n_1) = f(n_2) \rightarrow \frac{n_1}{\sqrt{n_1^2-1}} = \frac{n_2}{\sqrt{n_2^2-1}} \rightarrow \frac{n_1^2}{n_1^2-1} = \frac{n_2^2}{n_2^2-1} \rightarrow n_1^2 n_2^2 - n_1^2 = n_1^2 n_2^2 - n_2^2$ $\Rightarrow n_1^2 = n_2^2 \rightarrow n_1 = \pm n_2 \xrightarrow{\text{تابع یک به یک هست}} n = \frac{y}{\sqrt{y^2-1}} \rightarrow n^2 = \frac{y^2}{y^2-1}$ $\left[\begin{aligned} y &= \sqrt{\frac{y^2}{n^2-1}} \\ y^2 &= \frac{y^2}{n^2-1} \end{aligned} \right]$	1
$f^{-1}(n) = \frac{n}{1+ n }$ $g(n) = \sqrt{n} - 1$ $g^{-1}(-\frac{1}{2}) \Rightarrow g(n) = \frac{1}{2} \Rightarrow \sqrt{n} - 1 = \frac{1}{2} \Rightarrow n = \frac{9}{4} \Rightarrow g^{-1}(-\frac{1}{2}) = \frac{9}{4}$ $f(\frac{1}{2}) \Rightarrow f^{-1}(n) = \frac{1}{2} \Rightarrow \frac{n}{1+ n } = \frac{1}{2} \xrightarrow{\text{عکس}} 2n = -2 + n \Rightarrow n = \frac{-2}{3} \Rightarrow f(\frac{1}{2}) = \frac{-2}{3}$ $\Rightarrow f(\frac{1}{2}) + g^{-1}(-\frac{1}{2}) = \frac{-2}{3} + \frac{9}{4} = \frac{-23}{12} \Rightarrow \boxed{\frac{-23}{12}}$	9
$f^{-1}(n) = \sqrt[3]{n-1}$ $g(n) = f(n) + \sqrt{f(n)}$ $g^{-1}(12) \Rightarrow g(n) = 12 \Rightarrow f(n) + \sqrt{f(n)} - 12 = 0 \xrightarrow{\text{عکس}} \sqrt{f(n)} = 3 \Rightarrow f(n) = 9$ $\Rightarrow f(n) = 9 \Rightarrow f^{-1}(9) = \sqrt[3]{9-1} = 2 \Rightarrow \boxed{g^{-1}(12) = 2}$	