

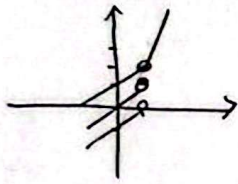
الف) $f_2 \{ (3, 2), (n, 5), (3, n^2 - n), (m, 2), (-1, 4) \}$

$\Rightarrow n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow \begin{cases} n = 2 \\ n = -1 \end{cases} \rightarrow \boxed{m = 3}$

ب) $f_2 \{ (-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3) \}$

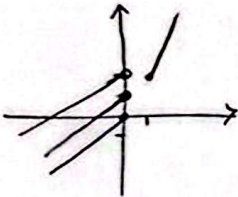
$\Rightarrow \boxed{m = 2} \rightarrow \boxed{a = -1} \rightarrow \boxed{n = 2}$

$f(n) = \begin{cases} 3n-1 & ; n \geq 1 \\ n+a & ; n < 1 \end{cases} \rightarrow \begin{cases} f(1) = 2 \\ f(1) = a+1 \end{cases}$



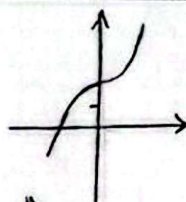
$\Rightarrow a+1 \leq 2 \rightarrow \boxed{|a| \leq 1}$

$f(n) = \begin{cases} 3n-1 & ; n \geq 1 \\ a+2n-1 & ; n < 1 \end{cases} \rightarrow \begin{cases} f(1) = 2 \\ f(0) = a-1 \end{cases}$



$\Rightarrow \begin{cases} a \geq 0 \\ a-1 < 2 \rightarrow a < 3 \end{cases} \Rightarrow \boxed{0 \leq a < 3}$

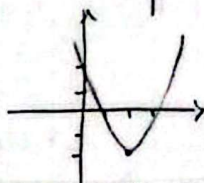
الف) $y = x^3 + 2$



$\Rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} \begin{cases} x = y^3 + 2 \\ \Rightarrow \boxed{y = \sqrt[3]{x-2}} \end{cases}$

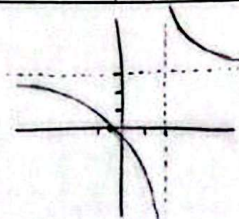
ب) $x^2 - 4x + 2$

$\left| \frac{-b}{2a} \right| = 2$
 $4 - 4 + 2 = 2$



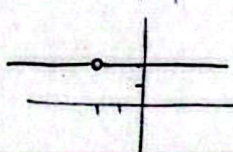
$\Rightarrow$ تابع یک به یک نیست

الف) $y = \frac{x+1}{x-2}$

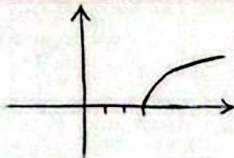
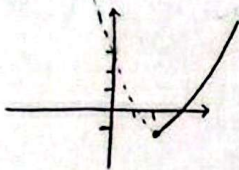


$\Rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} \begin{cases} x = \frac{y+1}{y-2} \\ \Rightarrow \boxed{y = \frac{x+1}{x-2}} \end{cases}$

ب) $y = \frac{x+2m}{m+2}$



$\Rightarrow$ تابع یک به یک نیست

الف) $y = \sqrt{n-3}$  $\Rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} n = \sqrt{y-3} \rightarrow y = n^2+3$ $n \in [0, +\infty)$	9
ب) $y = n^2 - 4n + 3, n \in [2, +\infty)$ $\frac{-b}{2a} = 2$ $f(2) = -1+3 = 2$  $\Rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} n = \sqrt{y-1} + 2$ $\Rightarrow n = (y-1)^2 - 1 \rightarrow y = \sqrt{n+1} + 2$	9
$f(n) = n + \sqrt{n} \xrightarrow{\text{عکس}} n = y + \sqrt{y + \frac{1}{4}} - \frac{1}{4} \rightarrow n = (y + \frac{1}{4})^2 - \frac{1}{4} \rightarrow y = (\sqrt{n + \frac{1}{4}} - \frac{1}{4})^2$ $\Rightarrow y = n + \frac{1}{4} - \sqrt{n + \frac{1}{4}} = a + b - \sqrt{n - c} \rightarrow$ $a = 1$ $b = \frac{1}{4}$ $c = -\frac{1}{4}$ $\Rightarrow a + b + c = 1 + \frac{1}{4} - \frac{1}{4} = 1$	9
$y = \frac{n}{\sqrt{n^2-1}}$ $\xrightarrow{\text{تعریف}} f(n_1) = f(n_2) \rightarrow \frac{n_1}{\sqrt{n_1^2-1}} = \frac{n_2}{\sqrt{n_2^2-1}} \rightarrow \frac{n_1^2}{n_1^2-1} = \frac{n_2^2}{n_2^2-1} \rightarrow n_1^2 n_2^2 - n_1^2 = n_1^2 n_2^2 - n_2^2$ $\Rightarrow n_1^2 = n_2^2 \rightarrow n_1 = \pm n_2 \rightarrow$ تابع یک به یک هست $\xrightarrow{\text{عکس}} n = \frac{y}{\sqrt{y^2-1}} \rightarrow n^2 = \frac{y^2}{y^2-1}$ $\left[\begin{aligned} y &= \sqrt{\frac{y^2}{n^2-1}} \leftarrow y = \sqrt{\frac{y^2}{n^2-1}} \leftarrow y^2 = \frac{y^2}{n^2-1} \end{aligned} \right]$	1
$f^{-1}(n) = \frac{n}{1+ n }$ $g(n) = \sqrt{n} - 1$ $g^{-1}(-\frac{1}{2}) \Rightarrow g(n) = \frac{1}{2} \Rightarrow \sqrt{n} - 1 = \frac{1}{2} \Rightarrow n = \frac{9}{4} \Rightarrow g^{-1}(-\frac{1}{2}) = \frac{9}{4}$ $f(\frac{1}{2}) \Rightarrow f^{-1}(n) = \frac{1}{2} \Rightarrow \frac{n}{1+ n } = \frac{1}{2} \xrightarrow{\text{عکس}} 2n = -2 + n \Rightarrow n = \frac{-2}{3} \Rightarrow f(\frac{1}{2}) = \frac{-2}{3}$ $\Rightarrow f(\frac{1}{2}) + g^{-1}(-\frac{1}{2}) = \frac{-2}{3} + \frac{9}{4} = \frac{-2}{3} + \frac{9}{4} = \frac{-8}{12} + \frac{27}{12} = \frac{19}{12}$	9
$f^{-1}(n) = \sqrt[3]{n-1}$ $g(n) = f(n) + \sqrt{f(n)}$ $g^{-1}(12) \Rightarrow g(n) = 12 \Rightarrow f(n) + \sqrt{f(n)} - 12 = 0 \Rightarrow \sqrt{f(n)} = 4 \Rightarrow f(n) = 16$ $\Rightarrow f(n) = 16 \Rightarrow f^{-1}(9) = \sqrt[3]{9-1} = 2 \Rightarrow g^{-1}(12) = 2$	10