تکالیف [illegible] ریاضی

[illegible] C

① الف) $f=\{(3,2),(x,5),(3,x^2-x),(m,2),(-1,4)\}$

$x^2-x=2 \to x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,\ m=3$; $x=-1$ غ ق ق

ب) $f=\{(-1,1),(1,2),(2,3),(a,m-1),(a+2,n),(m,3)\}$

$m=2 \to m-1 \to 2-1=1 \quad (a,1) \Rightarrow a=-1$

$a=-1 \to a+2 \to -1+2=1 \quad (1,n) \Rightarrow n=2$

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② $f(x)=\begin{cases} 3x-1 & x\ge 1 \\ x+a & x<1 \end{cases}$

$x=1 \to x+a\le 2$

$1+a\le 2$

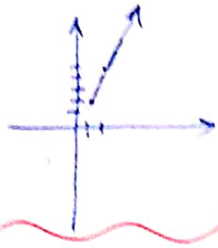
$a\le 1$

[illegible]

$x+a$ [illegible] $(-\infty,a+1)$

$1+a=2$

$a=1$



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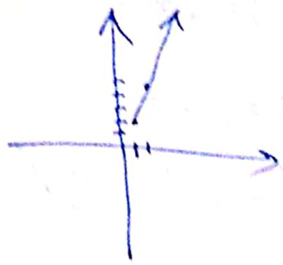
③ $f(x)=\begin{cases} 3x-1 & ,x\ge 1 \\ ax+a-1 & x\le 0 \end{cases}$

برد $3x-1 \leftarrow [2,+\infty)$ است برای اینکه تابع یک به یک [illegible] شیب [illegible] [illegible] ۲ [illegible]

$\begin{cases} x>0 \\ x-1<2 \to x<3 \end{cases} \Rightarrow 0<x<3$



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④ الف) $y=x^3+2 \quad y_1=y_2 \to x_1=x_2$

$x_1^3+2=x_2^3+2 \to x_1=x_2$

$x^3=y-2$

$x=\sqrt[3]{y-2} \to y=\sqrt[3]{x-2}$

$f^{-1}(x)=\sqrt[3]{x-2}$

ب) $x^2-4x+2 \to y_1=y_2 \to x_1=x_2$

$x_1^2-4x_1+2=x_2^2-4x_2+2$

$x_1\ne x_2 \to$ نیست

(5)

الف) $y=\frac{3x+1}{x-2}$

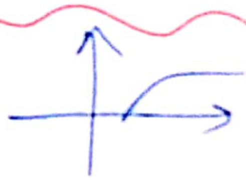
$yx-2y=3x+1 \to x(y-3)=2y+1$

$x=\frac{2y+1}{y-3} \to y=\frac{2x+1}{x-3}$ $\quad f^{-1}(x)=\frac{2x+1}{x-3}$

ب) $y=\frac{4+2x}{x+2}$ $\quad \frac{ax+b}{cx+d} \to ad=bc$ $\quad 2\times2=4\times1$ →

(6)

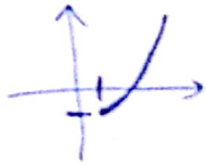
الف) $y=\sqrt{x-3}$



$x=\sqrt{y-3}$

$x^2=y-3 \to y=x^2+3 \;\; x\ge0$ $\quad f^{-1}(x)=x^2+3 \to x\ge0$

ب) $y=x^2-4x+3$, $x\in[2,+\infty)$



$y=(x-2)^2-1 \to x-2=\pm\sqrt{y+1}$

$x=\pm\sqrt{y+1}+2$ $\quad f^{-1}(x)=2+\sqrt{x+1}$

(ج)

$y=x+\sqrt{x} \to y=(\sqrt{x}+\frac{1}{2})^2-\frac{1}{4}$

$x=(\sqrt{y}+\frac{1}{2})^2-\frac{1}{4} \to x+\frac{1}{4}=(\sqrt{y}+\frac{1}{2})^2 \to \sqrt{y}=\sqrt{x+\frac{1}{4}}-\frac{1}{2}$

توان ۲ → $y=x+\frac{1}{4}+\frac{1}{4}-\sqrt{x+\frac{1}{4}} \to f^{-1}(x)=x+\frac{1}{2}-\sqrt{x+\frac{1}{4}} \to a=1 \;\; b=\frac{1}{2} \;\; c=-\frac{1}{4}$

$1+2(\frac{1}{2})+4(-\frac{1}{4})=1$

(8)

$$f(x_1) = f(x_2) \rightarrow \frac{x_1}{\sqrt{x_1^2 - 2}} = \frac{x_1}{\sqrt{x_2^2 - 2}}$$

$$\frac{x_1^2}{x_1^2 - 2} = \frac{x_2^2}{x_2^2 - 2}$$

$$x_1^2 x_2^2 - 2x_1^2 = x_2^2 x_1^2 - 2x_2^2 \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2 \rightarrow x_1 = x_2$$

$$y = \frac{x}{\sqrt{x^2 - 2}} \Rightarrow x = \frac{y}{\sqrt{y^2 - 2}} \rightarrow x^2 = \frac{y^2}{y^2 - 2} \rightarrow x^2 y^2 - 2x^2 = y^2$$

$$y^2 x^2 - y^2 = 2x^2$$

$$y^2 = \frac{2x^2}{x^2 - 1} \rightarrow y = \pm\sqrt{\frac{2x^2}{x^2 - 1}}$$

$$y = \frac{\sqrt{2}\, x}{\sqrt{x^2 - 1}}$$

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(9)

$$f^{-1}(x) = \frac{x}{1 + |x|} \quad g \text{ وارون} = -\tfrac{2}{5} \rightarrow \quad R_g = -\tfrac{2}{5}$$

$$g(x) = \sqrt{x} - 1$$

$$\sqrt{x} - 1 = -\tfrac{2}{5} \Rightarrow \sqrt{x} = \tfrac{3}{5} \rightarrow x = \tfrac{9}{25}$$

$$f(-\tfrac{2}{5}) + g^{-1}(-\tfrac{2}{5}) \qquad D_f = -\tfrac{2}{5} \;,\; R_{f^{-1}(x)} = -\tfrac{2}{5}$$

$$-\tfrac{2}{3} + \tfrac{9}{25} = \boxed{-\tfrac{23}{75}}$$

$$\frac{x}{1 + |x|} = -\tfrac{2}{5} \rightarrow 5x = -2 - 2|x|$$

$$5x = -2 + 2x$$

$$3x = -2 \rightarrow x = -\tfrac{2}{3}$$

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(10)

$$f^{-1}(x) = \sqrt[3]{x - 1}$$

$$g(x) = f(x) + \sqrt{f(x)}$$

$$g^{-1}(12) = ?$$

$$R_g = 12$$

$$y = \sqrt[3]{x - 1} \rightarrow x = \sqrt[3]{y - 1} \rightarrow x^3 = y - 1$$

$$x^3 + 1 = y$$

$$f(x) = x^3 + 1$$

$$g(x) = f(x) + \sqrt{f(x)} = 12$$

$$f(x) = 9 \rightarrow x^3 + 1 = 9 \rightarrow x^3 = 8 \rightarrow \boxed{x = 2}$$