تکالیف ۱۰ ... نازنین رمضان (۲۰)

(1)

الف) $f=\{(3,2),(x,5),(3,x^2-x),(m,2),(-1,4)\}$

$x^2-x=2 \to x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,\ m=3$ ، $x=-1$ غ ق ق

ب) $f=\{(-1,1),(1,2),(2,3),(a,m-1),(a+2,n),(m,3)\}$ (۵)

$m=2 \to m-1 \to 2-1=1 \quad (a,1) = a=-1$

$a=-1 \to a+2 \to -1+2=1 \quad (1,n) \Rightarrow n=2$

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(2)

$f(x)=\begin{cases}3x-1 & x\ge 1\\ x+a & x<1\end{cases}$

$x=1 \to x+a<2$

$1+a\le 2$

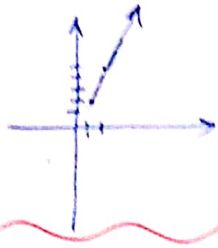
$a\le 1$

طبق یک به یک ... $(m,m+a)$ ... $m+a$

$1+a=2$

$a=1$

(۵)



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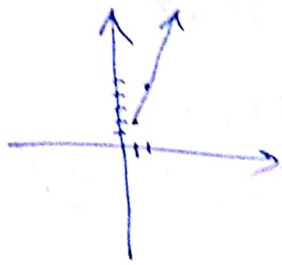
(3)

$f(x)=\begin{cases}3x-1 & ,x\ge 1\\ ax+a-1 & x\le 0\end{cases}$

برد $3x-1$ ← $[2,+\infty)$ است ... یک به یک ... شیب ... نقاط ...

(۵)

$\begin{cases}x>0\\ x-1<2 \to x<3\end{cases}$ ∩ $0<x<3$



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(4)

الف) $y=x^3+2 \quad y_1=y_2 \to x_1=x_2 \quad x^3=y-2$

$x_1^3+2=x_2^3+2 \to x_1=x_2 \quad x=\sqrt[3]{y-2} \to y=\sqrt[3]{x-2}$

$f^{-1}(x)=\sqrt[3]{x-2}$ (۵)

ب) $x^2-4x+2 \to y_1=y_2 \to x_1=x_2$

$x_1^2-4x_1+2=x_2^2-4x_2+2$

$x_1\ne x_2 \to$ یک به یک نیست

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الف) $y=\frac{3x+1}{x-2}$

$yx-2y=3x+1 \to x(y-3)=2y+1$

$x=\frac{2y+1}{y-3} \to y=\frac{2x+1}{x-3}$ $\quad f^{-1}(x)=\frac{2x+1}{x-3}$

ب) $y=\frac{4+2x}{x+2}$ $\quad \frac{ax+b}{cx+d} \to ad=bc$ $\quad 2\times2=4\times1$ → ثابت

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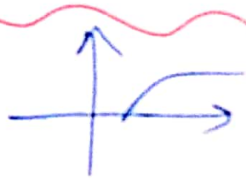
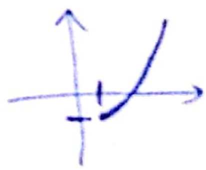
الف) $y=\sqrt{x-3}$



$x=\sqrt{y-3}$

$x^2=y-3 \to y=x^2+3 \quad x\ge 0$ $\quad f^{-1}(x)=x^2+3 \to x\ge 0$

ب) $y=x^2-4x+3$ $\quad , x\in[2,+\infty)$



$y=(x-2)^2-1 \to x-2=\pm\sqrt{y+1}$

$x=\pm\sqrt{y+1}+2$ $\quad f^{-1}(x)=2+\sqrt{x+1}$

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$y=x+\sqrt{x} \to y=(\sqrt{x}+\frac{1}{2})^2-\frac{1}{4}$

$x=(\sqrt{y}+\frac{1}{2})^2-\frac{1}{4} \to x+\frac{1}{4}=(\sqrt{y}+\frac{1}{2})^2 \to \sqrt{y}=\sqrt{x+\frac{1}{4}}-\frac{1}{2}$

توان ۲ → $y=x+\frac{1}{4}+\frac{1}{4}-\sqrt{x+\frac{1}{4}} \to f^{-1}(x)=x+\frac{1}{2}-\sqrt{x+\frac{1}{4}} \to a=1 \quad b=\frac{1}{2} \quad c=-\frac{1}{4}$

$1+2(\frac{1}{2})+4(-\frac{1}{4})=1$

(8)

$$f(x_1)=f(x_2)\rightarrow \frac{x_1}{\sqrt{x_1^2-2}}=\frac{x_1}{\sqrt{x_2^2-2}}$$

$$\frac{x_1^2}{x_1^2-2}=\frac{x_2^2}{x_2^2-2}$$

$$x_1^2x_2^2-2x_1^2=x_2^2x_1^2-2x_2^2\rightarrow x_1^2=x_2^2\rightarrow x_1=\pm x_2\rightarrow x_1=x_2$$

$$y=\frac{x}{\sqrt{x^2-2}}\Rightarrow x=\frac{y}{\sqrt{y^2-2}}\rightarrow x^2=\frac{y^2}{y^2-2}\rightarrow x^2y^2-2x^2=y^2$$

$$y^2x^2-y^2=2x^2$$

$$y^2=\frac{2x^2}{x^2-1}\rightarrow y=\pm\sqrt{\frac{2x^2}{x^2-1}}$$

$$y=\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}$$

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$$f^{-1}(x)=\frac{x}{1+|x|}\quad g\ \text{(ناحیه) بازدارنده}=-\frac{2}{5}\rightarrow \quad R_g=-\frac{2}{5}$$

$$g(x)=\sqrt{x}-1$$

$$\sqrt{x}-1=-\frac{2}{5}\Rightarrow \sqrt{x}=\frac{3}{5}\rightarrow x=\frac{9}{25}$$

(9)

$$f\left(-\frac{2}{5}\right)+g^{-1}\left(-\frac{2}{5}\right)\qquad D_f=-\frac{2}{5}\ ,\ R_{f^{-1}(x)}=-\frac{2}{5}$$

$$-\frac{2}{3}+\frac{9}{25}=-\frac{23}{75}$$

$$\frac{x}{1+|x|}=-\frac{2}{5}\rightarrow 5x=-2-2|x|$$

$$5x=-2+2x$$

$$3x=-2\rightarrow x=-\frac{2}{3}$$

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$$f^{-1}(x)=\sqrt[3]{x-1}$$

(10)

$$g(x)=f(x)+\sqrt{f(x)}\qquad y=\sqrt[3]{x-1}\rightarrow x=\sqrt[3]{y-1}\rightarrow x^3=y-1$$

$$x^3+1=y$$

$$g^{-1}(12)=?\qquad f(x)=x^3+1$$

$$g(x)=f(x)+\sqrt{f(x)}=12$$

$$R_g=12$$

$$f(x)=9\rightarrow x^3+1=9\rightarrow x^3=8\rightarrow x=2$$

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