

$$1) f = \{(1, 2), (2, 5), (3, n^2 - n), (m, 2), (-1, 4)\}$$

$$n^2 - n - 2 \rightarrow n^2 - n - 2 = 0$$

$$2 \rightarrow (n-2)(n+1) = 0 \rightarrow n = 2 \text{ یا } n = -1$$

$$\rightarrow m = 2$$

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$$2) f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3)\}$$

$$m = 2 \rightarrow m-1 = 2-1 = 1$$

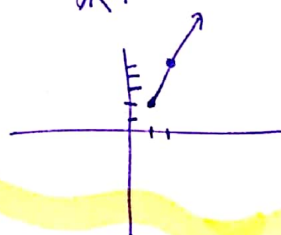
$$a = -1 \rightarrow a+2 = 1$$

$$-1 + 2 = 1$$

$$(a, 1) \rightarrow \overline{a = -1}$$

$$(1, m) \rightarrow \overline{m = 2}$$

$$f(n) = \begin{cases} 3n-1 & n \geq 1 \\ n+9 & n < 1 \end{cases}$$



$$n \geq 1 \rightarrow n \leq 2$$

$$1 \leq a \leq 2$$

$$a \leq 1$$

برای ۱ و ۲ بودن

به صورت $(-\infty, +\infty)$

$$1 \leq a \leq 2 \rightarrow a \leq 1$$

سوال ۲

$$f(n) = \begin{cases} 3n-1 & n \geq 1 \\ a(n+1) & n \leq 0 \end{cases}$$

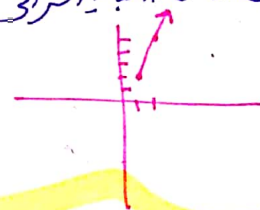
سوال ۳ - حد $a = ?$

$$\lim_{n \rightarrow \infty} (3n-1) = +\infty$$

برای این که تابع یک به یک باشد پس باید $a \neq 0$

$$\begin{cases} a > 0 \\ n-1 < 4 \rightarrow n < 5 \rightarrow 0 < a < 5 \end{cases}$$

این دو شرط باید اشتراک داشته باشند



$$1) y = n^3 + 2 \rightarrow y_1 = y_2 \rightarrow n_1 = n_2$$

$$n_1^3 + 2 = n_2^3 + 2 \rightarrow n_1^3 = n_2^3 \rightarrow n_1 = n_2$$

$$y = n^3 + 2 \rightarrow n = \sqrt[3]{y-2}$$

$$n = \sqrt[3]{y-2} \rightarrow y = \sqrt[3]{n-2}$$

$$f^{-1}(n) = \sqrt[3]{n-2}$$

سوال ۴

$$2) y = f(n+2) \rightarrow y_1 = y_2 \rightarrow n_1 = n_2$$

$$n_1^3 - f(n_1) + 2 = n_2^3 - f(n_2) + 2 \rightarrow n_1 = n_2$$

پس یک به یک نیست

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الف) $y = \frac{2x+1}{x-2}$

$yx - 2y = 2x + 1 \quad x(y-2) = 2y+1$

(سوال 5)

$x = \frac{2y+1}{y-2} \rightarrow y = \frac{2x+1}{x-2} \rightarrow f^{-1}(x) = \frac{2x+1}{x-2}$

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ب) $y = \frac{2x+1}{x-2}$

$\frac{ax+b}{cx+d} \rightarrow ad \neq bc$
 $2 \times 2 \neq 1 \times 1$

تابع همگراف است

(سوال 6)

الف) $y = \sqrt{x-2} \quad x = \sqrt{y+2} \rightarrow x^2 = y+2 \rightarrow y = x^2 - 2$

$f^{-1}(x) = x^2 - 2 \rightarrow x \geq 0$



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ب) $y = x^2 - 2, x \in [2, +\infty)$

$y = (x-2)^2 - 1 \rightarrow x-2 = \pm \sqrt{y+1} \rightarrow x = \pm \sqrt{y+1} + 2$

$f^{-1}(x) = 2 + \sqrt{x+1}$



$y = x + \sqrt{x} \rightarrow y = (\sqrt{x} + \frac{1}{4})^2 - \frac{1}{4} \rightarrow x = (\sqrt{y} + \frac{1}{4})^2 - \frac{1}{4}$

(سوال 7)

$x + \frac{1}{4} = (\sqrt{y} + \frac{1}{4})^2 \rightarrow \sqrt{y} = \sqrt{x + \frac{1}{4}} - \frac{1}{4}$

2 توان $\rightarrow y = x + \frac{1}{4} + \frac{1}{4} - \sqrt{x + \frac{1}{4}} \rightarrow f^{-1}(x) = x + \frac{1}{4} - \sqrt{x + \frac{1}{4}}$

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$1 + 2(\frac{1}{4}) + 2(-\frac{1}{4}) = 1$

$f(x_1) = f(x_2) \rightarrow \frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \rightarrow \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2}$

(سوال 8)

$x_1^2 x_2^2 - 2x_1^2 = x_2^2 x_1^2 - 2x_2^2 \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow x = \frac{y}{\sqrt{y^2-2}} \rightarrow x^2 = \frac{y^2}{y^2-2} \rightarrow y^2 = \frac{2x^2}{x^2-1} \rightarrow y = \pm \sqrt{\frac{2x^2}{x^2-1}}$

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$$f^{-1}(x) = \frac{x}{1+|x|}$$

$$g \text{ مقلوبه } = \frac{-x}{\delta} \rightarrow y \rightarrow \frac{-x}{\delta}$$

(سوال ۱)

$$\sqrt{x} - 1 = \frac{-x}{\delta} \rightarrow \sqrt{x} = \frac{x}{\delta} \rightarrow x = \frac{9}{8}$$

$$\frac{-x}{\delta} = \text{مقلوب } f \rightarrow \frac{-x}{\delta} = f^{-1}(x)$$

$$g(x) = \sqrt{x} - 1$$

$$f\left(\frac{-x}{\delta}\right) + g^{-1}\left(\frac{-x}{\delta}\right) = 0$$

$$\frac{-x}{\delta} + \frac{9}{8} = \frac{-x^2}{\sqrt{8}}$$

$$\frac{x}{1+|x|} = \frac{-x}{\delta} \rightarrow \delta x = -x - x|x|$$

$$\delta x = -x + x|x|$$

$$x = -x \rightarrow x = \frac{-x}{x}$$

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$$f^{-1}(x) = \sqrt[3]{x-1}$$

$$g(x) = f(x) + \sqrt{f(x)}$$

$$g^{-1}(12) = ?$$

$$g(x) = f(x) + \sqrt{f(x)} = 12$$

$$y = \sqrt[3]{x-1} \rightarrow x = \sqrt[3]{y-1} \rightarrow x^3 = y-1$$

$$f(x) = x^3 + 1 \leftarrow x^3 + 1 = y$$

$$x^3 + 1 = 9 \rightarrow x^3 = 8 \rightarrow x = 2$$

$$\boxed{x=2}$$

(سوال ۱۰)

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