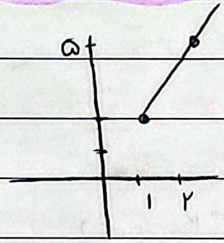
تکلیف شماره ۱۰ — دنیا جمشیدی — یازدهم تجربی

۱) الف) $m = 3$ $n^2 - 2n = 2$ $n^2 - 2n - 2 = 0$ $\Delta = 4 - \overset{-8}{\cancel{4(-2)}} = 12$

$\dfrac{2 \pm 2\sqrt{3}}{2} = \underline{1 \pm \sqrt{3} = n}$

ب) $m = 2$ $a = -1$ $\underline{n = 2}$

۲) $f(x) = \begin{cases} 3x - 1 & ; x \geq 1 \\ x + a & ; x < 1 \end{cases}$



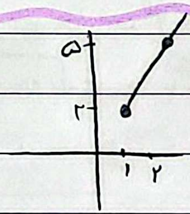
x	1
y	$1+a$

$1 + a \leq 2$

$\underline{a \leq 1}$

$[1, +\infty)$

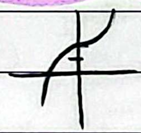
۳) $f(x) = \begin{cases} 3x - 1 & ; x \geq 1 \\ ax + a - 1 & ; x \leq 0 \end{cases}$



x	0
y	$a-1$

$a - 1 \leq 2$ $\underline{a \leq 3} \longrightarrow \underline{(-\infty, 3]}$

۴) الف) $y = x^3 + 2$



یک به یک

$x = y^3 + 2$

$y^3 = x - 2$

$\underline{y = \sqrt[3]{x - 2}}$

ب) $x^{2} - 4x + 2$

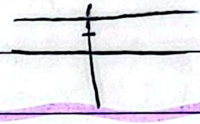
درجه ۲ ← سهمی ← یک به یک ✗

s.a.m

۵) الف) $y=\frac{3x+1}{x-2}$ هموگرافیک و یک به یک ✓ $x=\frac{3y+1}{y-2}$

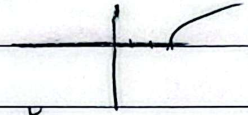
$xy-2x=3y+1$ $\quad (x-3)y=1+2x$ $\quad \boxed{y=\frac{1+2x}{x-3}}$

ب) $y=\frac{2x+4}{x+2}=2$ و $y=2$



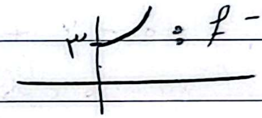
یک به یک X

۶) $y=\sqrt{x-3}\geq 0$



یک به یک ✓ $R_f=[0,+\infty)=D_{f^{-1}}$

$x=\sqrt{y-3}$ $\xrightarrow{(\ )^2}$ $x^2=y-3$ $\quad y=x^2+3;\ x\geq 0$

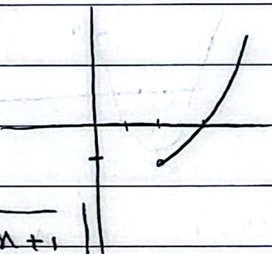


ب) $y=x^2-4x\overset{+4-1}{+3},\quad x\in[2,+\infty)$

$y=(x-2)^2-1$ $\quad (y-2)^2=x+1$

$x=(y-2)^2-1$ $\quad y-2=\pm\sqrt{x+1}$

$y=2\pm\sqrt{x+1}$ $\to$ $\boxed{y=2+\sqrt{x+1}}$



یک به یک ✓ $D_f=R_{f^{-1}}=[2,+\infty)$

۷) $f(x)=y=x+\sqrt{x}$ $\quad (0,+\infty)$ $\to$ $f^{-1}(x)=ax+b-\sqrt{x-c}$

$a+2b+4c$? $\quad y=(\sqrt{x}+\frac{1}{2})^2-\frac{1}{4}$ $\quad x=(\sqrt{y}+\frac{1}{2})^2-\frac{1}{4}$

$f^{-1}(x)=x+\frac{1}{2}-\sqrt{x+\frac{1}{4}}$ $\quad x+\sqrt{x}+\frac{1}{4}$ $\quad (\sqrt{y}+\frac{1}{2})^2=x+\frac{1}{4}$

$a=1\quad b=\frac{1}{2}\quad c=-\frac{1}{4}$ $\quad \sqrt{y}+\frac{1}{2}=\pm\sqrt{x+\frac{1}{4}}$

$$\begin{cases}\sqrt{y}=-\frac{1}{2}+\sqrt{x+\frac{1}{4}}\\ \sqrt{y}=-\frac{1}{2}-\sqrt{x+\frac{1}{4}}\end{cases}$$

$1+1+4(-\frac{1}{4})=1+1-1=1$ $\quad y=x+\frac{1}{4}+\frac{1}{4}-\sqrt{x+\frac{1}{4}}$

$\cancel{y=x+\frac{1}{4}+\frac{1}{4}+\sqrt{x+\frac{1}{4}}}$

s.a.m

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$y = \dfrac{x}{\sqrt{x^2-2}} \xrightarrow{y_1 = y_2} x_1 = x_2 \;?$

$\dfrac{x_1}{\sqrt{x_1^2-2}} = \dfrac{x_2}{\sqrt{x_2^2-2}} \rightarrow x_1\sqrt{x_2^2-2} = x_2\sqrt{x_1^2-2}$

$x_1^2(x_2^2-2) = x_2^2(x_1^2-2)$

$\cancel{x_1^2x_2^2} - 2x_1^2 = \cancel{x_2^2x_1^2} - 2x_2^2 \qquad \cancel{2}x_1^2 = \cancel{2}x_2^2 \qquad x_1 = \pm x_2$

x و y هم‌علامت ← $x_1 = x_2$ ← یک‌به‌یک ✓

$x = \dfrac{y}{\sqrt{y^2-2}} \qquad x\sqrt{y^2-2} = y \qquad x^2(y^2-2) = y^2$

$x^2y^2 - 2x^2 = y^2 \qquad (x^2-1)y^2 = 2x^2$

$y^2 = \dfrac{2x^2}{x^2-1} \qquad y = \pm\sqrt{\dfrac{2x^2}{x^2-1}} \xrightarrow{x,y\ \text{هم‌علامت}} y = \sqrt{\dfrac{2x^2}{x^2-1}}$

$f^{-1}(x) = \dfrac{x}{1+|x|} \qquad f(-\tfrac{2}{5}) + g^{-1}(-\tfrac{2}{5}) \quad \begin{cases} \dfrac{9}{25} - \dfrac{2}{7} = \dfrac{63-50}{175} = \dfrac{13}{175} \quad ۹ \\ \dfrac{9}{25} - \dfrac{2}{3} = \dfrac{27-50}{75} = -\dfrac{23}{75} \end{cases}$

$g(x) = \sqrt{x} - 1 \qquad \dfrac{x}{1+|x|} = -\dfrac{2}{5} \qquad 5x = -2 - 2|x| \quad \begin{cases} x \geq 0, & x = -\tfrac{2}{7} \\ x < 0 & x = -\tfrac{2}{3} \end{cases}$

$5x + 2|x| = -2$

$\sqrt{x} - 1 = -\dfrac{2}{5} \qquad \sqrt{x} = \dfrac{3}{5} \qquad x = \dfrac{9}{25}$

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$f^{-1}(x) = \sqrt[3]{x-1} \qquad g(x) = f(x) + \sqrt{f(x)} \qquad g^{-1}(12)\;?$

$f(x) + \sqrt{f(x)} = 12 \rightarrow f(x) = 9 \qquad x^3 + 1 = 9$

$y = \sqrt[3]{x-1} \rightarrow x^3 = y - 1 \qquad x^3 = 8$

$x = \sqrt[3]{y-1} \qquad y = x^3 + 1 \rightarrow f(x) = x^3 + 1 \qquad x = 2$

s.a.m