

الف) $f = \{(r, u) (r, v) (r, n^r - n) (m, r) (-1, r)\}$

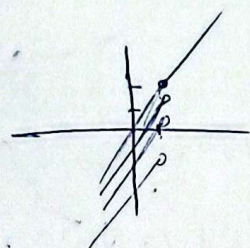
$n^r - n - r = 0$
 $(n-r)(n+1) = 0$
 $n = r \rightarrow (r, r)$
 $n = -1 \rightarrow (-1, 0) (-1, r)$

$(r, r)(m, r) \rightarrow m = r$

ب) $f = \{(-1, 1) (1, r) (r, r) (a, m-1) (a+r, n) (m, r)\}$
 $a = -1$
 $(a, 1) (a+r, n) (m, r)$
 $(-1, 1) (1, n) (m, r)$
 $m = r$

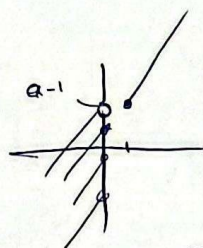
$(1, r) (1, r)$
 $n = r$

$f(n) = \begin{cases} r_{n-1} & n \geq 1 \rightarrow f(1) = r \\ n+a & n < 1 \\ f(1) = a+1 \end{cases}$



$a+1 \leq r$
 $a \leq 1$

$f(n) = \begin{cases} r_{n-1} & n \geq 1 \rightarrow f(1) = r \\ a_{n-1} + a - 1 & n \leq 0 \\ f(0) = a - 1 \end{cases}$



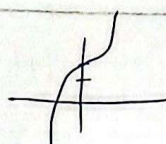
(1) $a \geq 0$

(2) $a-1 < r \rightarrow a < r$

(1) $a \geq 0$

$a \in (0, r)$

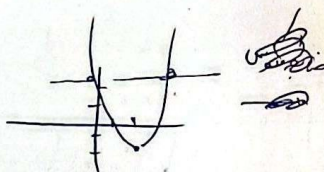
الف) $y = x^r + r$



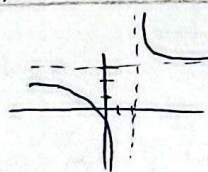
ضابطه مشتق

$x = y^r + r \rightarrow x - r = y^r \rightarrow y = \sqrt[r]{x-r}$

ب) $y = x^r - rx + r \rightarrow$ (X)
 $-\frac{b}{a} = \frac{r}{r} = 1$



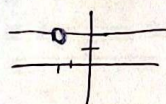
الف) $y = \frac{r_{n+1}}{n-r}$



ضابطه مشتق

$x = \frac{r_{y+1}}{y-r} = x \rightarrow y = -\frac{r_{x-1}}{x-r} = \frac{r_{x+1}}{x-r} = y$

ب) $y = \frac{r_{n+1}}{n+r} = \frac{r(n+r)}{(n+r)} \Rightarrow y = r$ (X)



الف) $y = \sqrt{x-3}$ ✓

ضابطه معکوس $x = \sqrt{y-3}$ $\xrightarrow{()^2}$ $x^2 = y-3$ $\Rightarrow y = x^2 + 3$
 $R(f) = [0, +\infty)$ $\xrightarrow{}$ $D(f^{-1}) = [0, +\infty)$

ب) $y = x^2 - 4x + 3, x \in [2, +\infty)$

ضابطه معکوس $x = \frac{y^2 - 4y + 3 + 1 - 1}{2} = \frac{(y-2)^2 - 1}{2} \Rightarrow \sqrt{x+1} + 2 = y$
 $R(f) = [-1, +\infty)$

$f(x) = x + \sqrt{x}$

$x \in (0, +\infty)$

$\Rightarrow f^{-1}(x) = ax + b - \sqrt{x-c}$

$y = x + \sqrt{x} + \frac{1}{2} - \frac{1}{2}$

$y = \left(\sqrt{x} + \frac{1}{2}\right)^2 - \frac{1}{2}$

ضابطه معکوس

$f^{-1}(x) \Rightarrow x = \left(\sqrt{y} + \frac{1}{2}\right)^2 - \frac{1}{2}$

$\left(\sqrt{x + \frac{1}{2}} - \frac{1}{2}\right)^2 = y$

$y = x + \frac{1}{2} + \frac{1}{2} - \sqrt{x + \frac{1}{2}}$

$y = x + \frac{1}{2} - \sqrt{x + \frac{1}{2}}$

$\frac{a=1}{b=\frac{1}{2}} \quad c=-\frac{1}{2}$

$\Rightarrow a+2b+c = 1+1-1 = 1$

$y = \frac{x}{\sqrt{x^2-2}}$

تعریف معکوس

$(x_1, y) \Rightarrow x_1 = x_2$

$y_1 = y_2$

$\frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \xrightarrow{()^2}$

$\frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2}$

ضابطه معکوس

$x = \frac{y}{\sqrt{y^2-2}} \xrightarrow{()^2}$

$\frac{y^2}{y^2-2} = x^2 \Rightarrow y^2 = x^2(y^2-2) \Rightarrow y^2 = x^2 y^2 - 2x^2 \Rightarrow y^2 - x^2 y^2 = -2x^2 \Rightarrow y^2(1-x^2) = -2x^2$

$y = \sqrt{\frac{2x^2}{x^2-1}}$

$y = \pm \sqrt{\frac{2x^2}{x^2-1}}$

تابع معکوس

$x_1^2 x_2^2 - 2x_1^2 = x_2^2 x_1^2 - 2x_2^2 \Rightarrow -2x_1^2 = -2x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$

و گویا این مجموعه دو بار مرتبه را پیدا می کند چون که با جابجایی x به دست می آید (مردن خروجی صواب است)

$x_1 = x_2$ یعنی $y_1 = y_2$

$f^{-1}(x) = \frac{x}{|x|+1} \rightarrow f\left(\frac{-2}{3}\right) = ? \rightarrow f^{-1}(?) = \frac{-2}{3} \rightarrow \frac{x}{|x|+1} = \frac{-2}{3} \rightarrow -2|x| - 2 = 3x$

$-2|x| = 3x + 2$

$g(x) = \sqrt{x} - 1 \rightarrow g^{-1}\left(\frac{-2}{3}\right) = ? \rightarrow g(?) = \frac{-2}{3}$

$\sqrt{x} - 1 = \frac{-2}{3} \Rightarrow \sqrt{x} = \frac{1}{3}$

$\frac{9}{25} - \frac{2}{5} = \frac{25-10}{25} = \frac{15}{25} = \frac{3}{5}$

$\sqrt{x} = \frac{1}{3} \Rightarrow x = \frac{1}{9}$

$x = \frac{3}{5}x + 1 \Rightarrow x - \frac{3}{5}x = 1 \Rightarrow \frac{2}{5}x = 1 \Rightarrow x = \frac{5}{2}$

$f^{-1}(x) = \sqrt[3]{x-1} \rightarrow f(x) = y = \sqrt[3]{x-1} \Rightarrow x = y^3 + 1 \Rightarrow f(x) = x^3 + 1$

$g(x) = f(x) + \sqrt{f(x)} \rightarrow g(x) = x^3 + 1 + \sqrt{x^3 + 1}$

$g^{-1}(12) = ? \rightarrow g(?) = 12$

$x^3 + 1 + \sqrt{x^3 + 1} = 12 \Rightarrow x^3 + 1 = 9 \Rightarrow x^3 = 8 \Rightarrow x = 2$

مردن صحیح