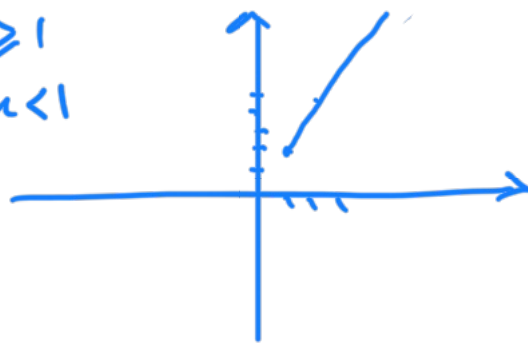


الف) $n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow \begin{cases} n = -1 \\ n = 2 \end{cases}$ $\sigma \in (-1, 2)$

$(2, n^2 - n) \xrightarrow{n=2} (2, 2) = (m, 2) \Rightarrow m = 2$

ب) $f \left\{ (-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 2) \right\}$
 $\underbrace{a = -1} \quad \underbrace{m = 2} \quad \underbrace{n = 2}$

$f(x) = \begin{cases} 2x-1; & x \geq 1 \\ x+a; & x < 1 \end{cases}$



$a \in (-\infty, 1]$

$f(x) = \begin{cases} 2x-1; & x \geq 1 \\ ax+a-1; & x < 1 \end{cases}$



$a = 0 \rightarrow a-1 < 2 \rightarrow a < 2$
 $a > 0 \rightarrow \dots \rightarrow 0 < a < 2$

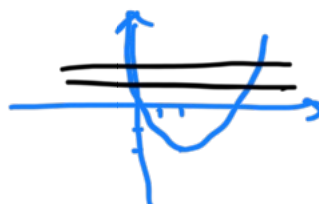
الف) $y = x^3 + 2 \rightarrow$



یک به یک

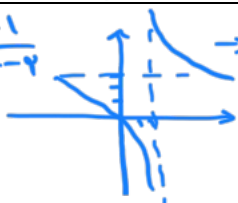
$x = y^3 + 2 \rightarrow y = \sqrt[3]{x-2}$

ب) $y = x^2 - 2x + 2 \rightarrow$
 $\min \left| \frac{2}{-2} \right|$



یک به یک

$y = \frac{2x+1}{x-2} \rightarrow$ $\rightarrow$ $\frac{2x+1}{x-2} = y \rightarrow 2x+1 = y(x-2) \rightarrow 2x+1 = xy-2y \rightarrow xy-2x = 2y+1 \rightarrow x(y-2) = 2y+1 \rightarrow x = \frac{2y+1}{y-2}$
 $\rightarrow xy-2y = 2x+1 \rightarrow y = \frac{2x+1}{x-2}$



$y = \frac{2x+1}{x-2} = \frac{2(x+2)}{(x+2)} = 2 \rightarrow$ یک به یک

$y = \sqrt{x-r}$
 $x = \sqrt{y-r} \xrightarrow{\text{تربيع}}$
 $y = x^2 + r$

$\Rightarrow y = x^2 - \varepsilon x + r \quad x \in [r, +\infty)$
 $\rightarrow y^2 - \varepsilon y + r - x = 0 \rightarrow y = \frac{\varepsilon \pm \sqrt{\varepsilon^2 - 4(r-x)}}{2} = \frac{\varepsilon \pm \sqrt{\varepsilon^2 + 4x}}{2} = r \pm \sqrt{x+1}$

$\rightarrow D_f^{-1} = R_f(x) \rightarrow [-r, +\infty) \rightarrow \text{مجال } D \text{ جزئي}$

6

$f(x) = x + \sqrt{x} \Rightarrow f'(x) = 1 + \frac{1}{2\sqrt{x}}$
 $\hookrightarrow f'(x) = \frac{1}{2} \Rightarrow x = y + \sqrt{y} + \frac{1}{2} - \frac{1}{2} = (\sqrt{y} + \frac{1}{2})^2 - \frac{1}{2} \Rightarrow x + \frac{1}{2} = (\sqrt{y} + \frac{1}{2})^2$
 $\Rightarrow \sqrt{y} + \frac{1}{2} = \sqrt{x + \frac{1}{2}} = \sqrt{y} \Rightarrow \sqrt{x + \frac{1}{2}} - \frac{1}{2} = \sqrt{y} \xrightarrow{\text{تربيع}} y = x + \frac{1}{2} + \frac{1}{2} - \sqrt{x + \frac{1}{2}} = x - \sqrt{x + \frac{1}{2}} + \frac{1}{2}$

$\Rightarrow a + \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2} \quad a=1, b=\frac{1}{2}, c=-\frac{1}{2}$

7

$y = \frac{x}{\sqrt{x^2-1}} \rightarrow \frac{x_1}{\sqrt{x_1^2-1}} = \frac{x_2}{\sqrt{x_2^2-1}} \xrightarrow{\text{تربيع}} \frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1} \Rightarrow$
 $x_1^2(x_2^2-1) = x_2^2(x_1^2-1) \Rightarrow x_1^2 x_2^2 - x_1^2 = x_1^2 x_2^2 - x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \rightarrow \text{دالة زوجية}$

$\Rightarrow x_1 = x_2 \Rightarrow \text{دالة زوجية} \quad \hookrightarrow x = \frac{y}{\sqrt{y^2-1}} \quad x^2 = \frac{y^2}{y^2-1} \Rightarrow x^2(y^2-1) = y^2 \Rightarrow y^2 = \frac{x^2}{x^2-1}$
 $\rightarrow y = \pm \sqrt{\frac{x^2}{x^2-1}} = \pm \frac{x}{\sqrt{x^2-1}} \quad y > 0 \Rightarrow y = \frac{x}{\sqrt{x^2-1}}$

8

$f(x) = \frac{x}{1+|x|} \quad g(x) = \sqrt{x} - 1 \quad f(-\frac{r}{2}) + g(-\frac{r}{2}) = -\frac{r}{2} + \frac{r}{2} = \frac{-r + r}{2} = \frac{-r + r}{2} = \frac{-r + r}{2}$

$y = \frac{x}{1+|x|} \rightarrow f(x) \rightarrow x = \frac{y}{1+|y|} \Rightarrow y(1+|y|) = x \Rightarrow y(1+y) = -x \Rightarrow y = \frac{x}{1-x}$
 $y + yx = x \rightarrow y(1+x) = x \Rightarrow y = \frac{x}{x+1}$

$f(x) = \begin{cases} \frac{x}{1-x} & x \geq 0 \\ \frac{x}{1+x} & x < 0 \end{cases} \rightarrow f(-\frac{r}{2}) = \frac{-r/2}{1-r/2} = \frac{-r}{2-r}$

$\rightarrow g^{-1}(x) \Rightarrow x = \sqrt{y} - 1 \quad x+1 = \sqrt{y} \rightarrow y = (x+1)^2 \rightarrow y = \frac{9}{16}$

9

$f(x) = \sqrt{x+1} \quad g(x) = f(x) + \sqrt{f(x)} \quad g^{-1}(x) = ?$
 $x = \sqrt{y+1} \xrightarrow{\text{تربيع}} x^2 = y+1 \Rightarrow y = x^2 - 1$
 $y^2 + 1 = x^2 - \sqrt{\varepsilon x + 1} - 1$

$g(x) = x^2 + 1 + \sqrt{x^2 + 1} \quad g^{-1}(x) = y^2 + 1 + \sqrt{y^2 + 1}$
 $y = t^2 + 1 \rightarrow t = \frac{-1 \pm \sqrt{y^2 + 1}}{2}$

$t \geq 0 \rightarrow t = \frac{-1 + \sqrt{y^2 + 1}}{2} = \sqrt{y^2 + 1} - 1$
 $y + 1 = \frac{y^2}{\varepsilon x + 1} + 1 - 1 \sqrt{\varepsilon x + 1}$

$y = \sqrt{\frac{y^2}{\varepsilon x + 1} - 1} \xrightarrow{\text{تربيع}} y^2 = \frac{y^2}{\varepsilon x + 1} - 1 \Rightarrow y^2(\varepsilon x + 1) = y^2 - \varepsilon x + 1$

10