

۲۰

سایه‌های استوانه - یازدهم دقت - شماره ۱۵

$$f = \{(r, r), (n, \omega), (r, n^2 - n), (m, r), (-1, r)\} \quad (1)$$

$$n^2 - n = r \rightarrow n^2 - n - r = 0 \rightarrow (n - r)(n + 1) = 0$$

$$\boxed{n = r}$$

$$\boxed{n = -1}$$

غ ق ق
چون ریشه یک به یک می‌آید

$$(r, n^2 - n) \xrightarrow{n=r} (r, r) = (m, r)$$

$$\boxed{m = r}$$

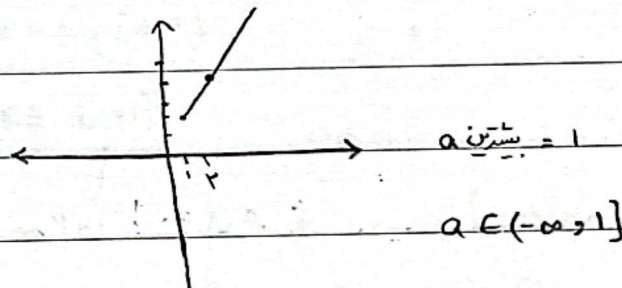
$$f = \{(-1, 1), (1, r), (r, r), (a, m-1), (a+r, n), (m, r)\}$$

$$\boxed{m = r}$$

$$\boxed{a = -1}$$

$$\boxed{n = r}$$

$$f(x) = \begin{cases} 3x - 1 & x \geq 1 \\ x + a & x < 1 \end{cases}$$

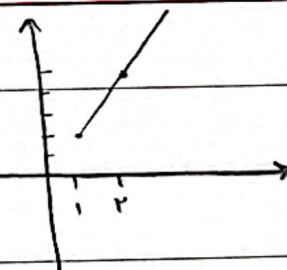


$$f(x) = \begin{cases} 3x - 1 & x \geq 1 \\ ax + a - 1 & x < 0 \end{cases}$$

$$x = 0$$

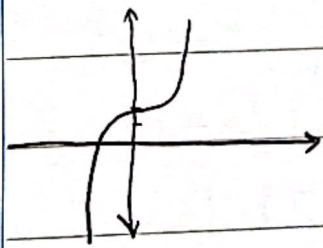
$$\boxed{a > 0}$$

$$a - 1 < r \rightarrow a < r \rightarrow 0 < a < r$$



$$\text{الف) } y = x^3 + 2$$

$$x = y^3 + 2$$



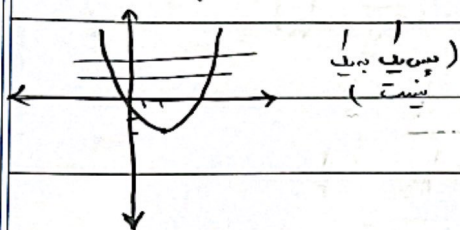
سین تابع یک به یک است

$$x - 2 = y^3$$

$$\sqrt[3]{x - 2} = \sqrt[3]{y^3} \Rightarrow \sqrt[3]{x - 2} = y$$

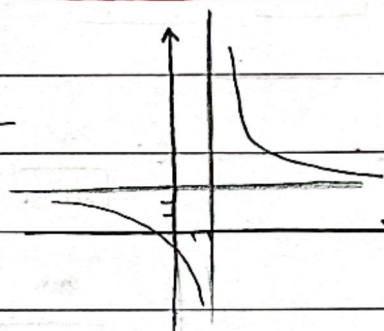
ب) $y = x^2 - 4x + 2$ $\Delta = 16 - 4(2)(1) = 8$

ext | $\frac{-b}{2a} = (2)$
 $\frac{-\Delta}{2a} = \frac{-8}{2} = (-2)$



الف) $y = \frac{x+1}{x-2}$

سایه بندی است



$x = \frac{y+1}{y-2}$
 $y = \frac{x+1}{x-2}$

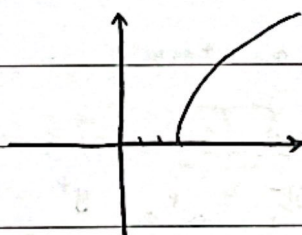
ب) $y = \frac{x+2}{x+2} \rightarrow \frac{x+2}{x+2} = \frac{2(x+2)}{(x+2)} = (2)$ سایه بندی است

الف) $y = \sqrt{x-3}$

$x = \sqrt{y-3}$

$x^2 = y-3$

$x^2 + 3 = y$



ب) $y = x^2 - 4x + 2$, $x \in [2, +\infty)$

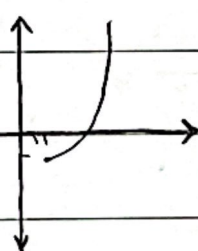
ext | $\frac{-b}{2a} = (2)$
 (-1)

$x = y^2 - 4y + 2$

$x = (y-2)^2 - 1$

$\sqrt{x+1} = y-2$

$\sqrt{x+1} + 2 = y$



Subo

$$f(x) = x + \sqrt{x} \rightarrow f(x)^{-1} = ax + b - \sqrt{x-c} \quad (v)$$

$$f(x)^{-1} \Rightarrow x = y + \sqrt{y} + \frac{1}{r} - \frac{1}{r} = \left(\sqrt{y} + \frac{1}{r}\right)^r - \frac{1}{r}$$

$$\rightarrow x + \frac{1}{r} = \left(\sqrt{y} + \frac{1}{r}\right)^r$$

$$\sqrt{y} + \frac{1}{r} = \sqrt{x + \frac{1}{r}} \rightarrow \sqrt{y} = \sqrt{x + \frac{1}{r}} - \frac{1}{r}$$

$$\text{röb} \rightarrow y = x + \frac{1}{r} + \frac{1}{r} - \sqrt{x + \frac{1}{r}} = x - \sqrt{x + \frac{1}{r}} + \frac{1}{r}$$

$$a=1$$

$$b=\frac{1}{r}$$

$$c=-\frac{1}{r}$$

$$a+rb+rc=0$$

$$y = \frac{x}{\sqrt{x^r - r}} \rightarrow y = \frac{x_1}{\sqrt{x_1^r - r}} \quad \left. \begin{array}{l} y = \frac{x_1}{\sqrt{x_1^r - r}} \\ y = \frac{x_1}{\sqrt{x_1^r - r}} \end{array} \right\} \rightarrow \frac{x_1}{\sqrt{x_1^r - r}} = \frac{x_1 r}{\sqrt{x_1^r - r}}$$

$$\text{röb} \rightarrow \frac{x_1^r}{x_1^r - r} = \frac{x_1^r}{x_1^r - r} \rightarrow x_1^r x_1^r - r x_1^r = x_1^r x_1^r - r x_1^r$$

$$\rightarrow x_1^r = x_1^r, x_1 = \pm x_1 \rightarrow \text{Gib} \quad \Leftarrow \text{wird es sein}$$

$$x_1 = x_1 \rightarrow \frac{1}{x_1}$$

$$x = \frac{y}{\sqrt{y^r - r}} \xrightarrow{\text{röb}} x^r = \frac{y^r}{y^r - r} \rightarrow x^r y^r - r x^r = y^r$$

$$y^r (x^r - 1) = r x^r$$

$$y^r = \frac{r x^r}{x^r - 1}$$

$$\rightarrow y = \pm \sqrt{\frac{r x^r}{x^r - 1}} = \pm \frac{x \sqrt{r}}{\sqrt{x^r - 1}} \xrightarrow{\text{wird es sein}} y = \frac{x \sqrt{r}}{\sqrt{x^r - 1}}$$

$$f^{-1}(x) = \frac{x}{1+|x|} \quad g(x) = \sqrt{x-1} \quad (9)$$

$$f\left(-\frac{r}{a}\right) + g^{-1}\left(-\frac{r}{a}\right) = \frac{-r}{r} + \frac{a}{ra} = \frac{-a^2 + r^2}{ra} = \frac{-r^2}{ra}$$

$$y = \frac{x}{1+|x|} \rightarrow f(x) \rightarrow x = \frac{y}{1+|y|} \rightarrow x + x|y| = y$$

$$y \geq 0 \rightarrow y(x-1) = -x \rightarrow y = \frac{x}{1-x}$$

$$y < 0 \rightarrow y + yx = x \rightarrow y(1+x) = x \rightarrow y = \frac{x}{x+1}$$

$$f(x) = \begin{cases} \frac{x}{1-x} & x \geq 0 \\ \frac{x}{x+1} & x < 0 \end{cases} \quad f\left(-\frac{r}{a}\right) = \frac{-r}{r}$$

$$g^{-1}(x) \rightarrow x = \sqrt{y-1} \quad x+1 = \sqrt{y} \quad y = (x+1)^2$$

$$y = \frac{a}{ra}$$

$$f(x)^{-1} = \sqrt[3]{x-1} \quad g(x) = f(x) + \sqrt{f(x)} \quad g^{-1}(x) = ? \quad (10)$$

$$x = \sqrt[3]{y-1} \quad x^3 = y-1 \rightarrow y = x^3+1$$

$$g(x) = x^3+1 + \sqrt{x^3+1} \quad g(x)^{-1} \rightarrow x = y^3+1 + \sqrt[3]{y^3+1}$$

$$x = t^3+t \rightarrow t = -1 \pm \sqrt{1+x}$$

$$t \geq 0 \rightarrow \sqrt{1+x} - 1$$

$$\rightarrow \sqrt[3]{y^3+1} = \sqrt[3]{1+x} - 1 \quad y^3+1 = 1+x+1 - \sqrt[3]{1+x}$$

$$y^3+1 = \frac{1+x - \sqrt[3]{1+x} + 1}{1} \rightarrow y^3 = \frac{1+x - \sqrt[3]{1+x} - 1}{1}$$

$$y = \sqrt[3]{\frac{1+x - \sqrt[3]{1+x} - 1}{1}} \quad g^{-1}(x) \rightarrow y = \sqrt[3]{\frac{x^3 - x - 1}{1}} = (11)$$

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