

(١٧)

$$1- A = \{1, 2, 3\} \quad B = \{2, 3\} \Rightarrow A \times B - B^2 = \{(1, 2)(2, 2)(3, 2)(1, 3)(2, 3)(3, 3)\} - \{(2, 2)(2, 3)(3, 2)(3, 3)\}$$

$$\Rightarrow A \times B - B^2 = \{(1, 2)(1, 3)\}$$

$$2- \text{ا) } f(n) = \{(n, m^2)(n, 1)(-n, m)(-n, m)(n, m+1)(m, \varepsilon)\}$$

$$\Rightarrow m^2 = m+1 \Rightarrow m^2 - m - 1 = 0 \Rightarrow (m+1)(m-2) = 0 \Rightarrow \begin{cases} m = -1 \\ m = 2 \end{cases} \rightarrow \{ \dots (2, 1) \dots (2, \varepsilon) \} \Rightarrow \text{مرفوض}$$

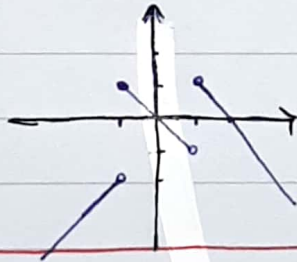
$$ب) g(n) = \{(n, m^2)(n, 1)(m, \varepsilon)(n, m+1)(-n, m)(-1, 2)\}$$

$$\Rightarrow m^2 = m+1 \xrightarrow{\text{طريقة المميز}} \begin{cases} m = -1 \\ m = 2 \end{cases} \rightarrow \{ \dots (2, \varepsilon) \dots (-1, 2) \} \Rightarrow \text{مرفوض}$$

$$3- f(x) = \begin{cases} x^2 - 1 & ; x \leq 1 \quad (1) \\ ax + b & ; 1 < x \leq 2 \quad (2) \\ x^2 & ; x > 2 \quad (3) \end{cases} \xrightarrow{(1) \text{ و } (2)} \begin{cases} 1 - 1 = a + b = 0 \\ a + b = 1 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = -1 \end{cases} \Rightarrow axb = -48$$

$$4- f(x) = \begin{cases} 2x - 3 & ; x \geq k \\ \varepsilon - 3x & ; x \leq k \end{cases} \xrightarrow{x=k} 2k - 3 = \varepsilon - 3k \Rightarrow k \geq \frac{\varepsilon}{5} \geq 1.2$$

$$5- f(x) = \begin{cases} -x + 2 & ; x > 1 \\ -x & ; -1 \leq x \leq 1 \\ x - 1 & ; x < -1 \end{cases}$$



$$\Rightarrow D_f = \mathbb{R} - \{1\} \\ R_f = (-\infty, 1]$$

$$6- \text{ا) } xy^2 + x^2 + 1 = 0 \rightarrow xy^2 = -x^2 - 1 \xrightarrow{x \neq 0} y^2 = -x - \frac{1}{x} \xrightarrow{y_1^2 = y_2^2} y_1^2 - y_2^2 = 0 \rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) = 0$$

$$\begin{cases} y_1 - y_2 = 0 \rightarrow y_1 = y_2 \checkmark \\ y_1^2 + y_1 y_2 + y_2^2 = 0 \end{cases}$$

$$\Delta \rightarrow y_1^2 - \varepsilon y_1^2 = -3y_1^2 < 0 \rightarrow \Delta < 0, a > 0 \rightarrow \text{لا يوجد حلا حقيقي} \Rightarrow \text{مرفوض}$$

$$ب) x^2 + \varepsilon y^2 - 2x - 14y + 10 = 0$$

$$L = (x^2 - 2x + 1) + (\varepsilon y^2 - 14y + 9) = (x-1)^2 + (\sqrt{\varepsilon}y - 7)^2 = 0 \Rightarrow \begin{cases} x = 1 \\ y = 7/\sqrt{\varepsilon} \end{cases} \checkmark$$

$$7- \text{ا) } x \sin y = y \sin x \xrightarrow{x=0} 0 \sin y = y \sin 0 \rightarrow y \sin y = 0 \Rightarrow y = 0 \text{ مرفوض}$$

$$ب) \left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} \right)^2 \rightarrow \frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{xy}{yn}} = \varepsilon \xrightarrow{xy \neq 0} \frac{x^2 + y^2}{xy} + 2 = \varepsilon \rightarrow x^2 + y^2 - 2xy = 0 \Rightarrow (x-y)^2 = 0 \Rightarrow x = y \neq 0 \checkmark$$

$$1. \text{ ا) } \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{3-x}{x}} \Rightarrow \begin{matrix} x-3 \neq 0 \Rightarrow x \neq 3 \\ x \neq 0 \end{matrix}$$

$$\frac{x-1}{x-3} \geq 0 \Rightarrow \frac{1}{x-3} \geq 0 \Rightarrow \{x \in (-\infty, 1] \cup (3, +\infty)\}$$

$$\frac{3-x}{x} \geq 0 \Rightarrow \frac{3}{x} \geq 1 \Rightarrow \{x \in (0, 3]\}$$

$$\Rightarrow D_f = (0, 1]$$

$$ب) \sqrt{x} + \sqrt{y-1} = \sqrt{3} \rightarrow x \geq 0 \Rightarrow D_f = [0, +\infty)$$

$$\sqrt{y-1} = \sqrt{x} \Rightarrow \sqrt{x} \leq \sqrt{y} \Rightarrow x \leq y$$

$$D_f = [0, 3]$$

$$9. \text{ ا) } y = \frac{\sqrt{x^2 - 5x + 4}}{x^2 - 10x + 14} \rightarrow \begin{matrix} (x-1)(x-4) \rightarrow \frac{1}{x-1} + \frac{4}{x-4} \\ (x-2)(x-1) \rightarrow \frac{1}{x-2} + \frac{1}{x-1} \end{matrix} \Rightarrow \frac{1}{x-1} + \frac{4}{x-4} \geq 0 \Rightarrow \{x \in (-\infty, 1] \cup (4, +\infty)\}$$

$$ب) y = \sqrt{\frac{x^2 - 5x + 4}{x^2 - 10x + 14}} \Rightarrow \frac{1}{x-1} + \frac{4}{x-4} \geq 0 \Rightarrow D_y = (-\infty, 1] \cup (4, +\infty)$$

$$10. \text{ ا) } y = \frac{\sqrt{x^2 - 6x + 3}}{\sqrt{x^2 - 5x + 8}} \xrightarrow{\text{مضروب}} x^2 - 6x + 3 = (x-2)(x-3) \Rightarrow \frac{1}{x-2} + \frac{3}{x-3} \geq 0$$

$$\xrightarrow{\text{مضروب}} x^2 - 5x + 8 = (x-4)(x-2) \Rightarrow \frac{1}{x-4} + \frac{4}{x-2} \geq 0$$

$$\Rightarrow D_y = (-\infty, 1) \cup [\frac{3}{2}, +\infty)$$

$$ب) y = \sqrt{\frac{x^2 - 6x + 3}{x^2 - 5x + 8}} \Rightarrow \frac{1}{x-2} + \frac{3}{x-3} \geq 0 \Rightarrow D_f = (-\infty, \frac{3}{2}) \cup [\frac{3}{2}, +\infty) - \{1\}$$