

$$A = \{1, 2, 3\}$$

$$B = \{2, 3\}$$

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$B^c = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$\Rightarrow A \times B - B^c = \{(1, 2), (1, 3)\}$$

$$\text{الف) } f(n) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 2)\}$$

$$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0 \rightarrow \begin{cases} m=2 \\ m=-1 \end{cases}$$

$$\text{ب) } g(n) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$$

$$m^2 = m+2 \rightarrow (m-2)(m+1) = 0 \rightarrow \begin{cases} m=2 \\ m=-1 \end{cases} \Rightarrow \text{هیچ گاه تابع نمی تواند باشد}$$

$$f(n) = \begin{cases} n^2 - 1 & ; n < 1 \\ an + b & ; 1 \leq n \leq 2 \\ n^3 & ; n \geq 2 \end{cases}$$

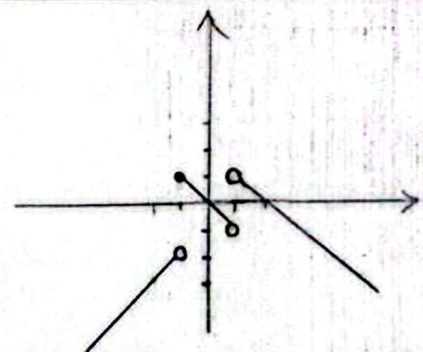
$$\begin{aligned} n=1 &\rightarrow 1^2 - 1 = a(1) + b \rightarrow a + b = 0 \\ n=2 &\rightarrow 2^2 = a(2) + b \rightarrow 2a + b = 4 \end{aligned} \Rightarrow \begin{cases} a=1 \\ b=-1 \end{cases}$$

$$f(n) = \begin{cases} 2n - 3 & ; n \geq K \\ 4 - 3n & ; n \leq K \end{cases}$$

$$n=K \rightarrow 2K - 3 = 4 - 3K \rightarrow 5K = 7 \rightarrow K = \frac{7}{5}$$

$$f(n) = \begin{cases} -n + 2 & ; n > 1 \\ -n & ; -1 \leq n \leq 1 \\ n - 1 & ; n < -1 \end{cases}$$

$$\begin{aligned} |D_f| &= \mathbb{R} - \{1\} \\ |R_f| &= (-\infty, 1] \end{aligned}$$



الف) $xy^2 + x^2 + 1 = 0 \Rightarrow \begin{cases} xy^2 = -x^2 - 1 \\ xy^2 = -x^2 - 1 \end{cases} \Rightarrow xy^2 = xy^2 \Rightarrow \boxed{xy = -1}$ ✓

ب) $x^2 + y^2 - 2x - 12y + 10 = 0 \Rightarrow (x^2 - 2x + 1) + (y^2 - 12y + 36) - 25 = 0 \Rightarrow (x-1)^2 + (y-6)^2 = 25 \Rightarrow \boxed{(1, 6)}$ ✓

الف) $x \sin y = y \sin x \xrightarrow{x=0} 0 = y \sin 0 \rightarrow y \in \mathbb{R}$ ✓

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{(\cdot)^2} \frac{x}{y} + \frac{y}{x} + 2 = 4 \Rightarrow \frac{x}{y} + \frac{y}{x} = 2 \Rightarrow \frac{x^2 + y^2}{xy} = 2 \Rightarrow x^2 + y^2 = 2xy \Rightarrow (x-y)^2 = 0 \Rightarrow \boxed{y = x}$ ✓

الف) $y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{x-2}{x}}$ $\frac{x-1}{x-3} \geq 0 \Rightarrow \frac{1}{x-3} \geq 0 \Rightarrow x > 3$ $\frac{x-2}{x} \geq 0 \Rightarrow \frac{1}{x} \geq 0 \Rightarrow x > 0$ $\Rightarrow D_f = (3, \infty)$ ✓

ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3}$ $x \geq 0, y-1 \geq 0 \Rightarrow y \geq 1$ $\sqrt{x} = \sqrt{3} - \sqrt{y-1} \Rightarrow \sqrt{x} \geq 0 \Rightarrow \sqrt{3} - \sqrt{y-1} \geq 0 \Rightarrow \sqrt{y-1} \leq \sqrt{3} \Rightarrow y-1 \leq 3 \Rightarrow y \leq 4$ $\Rightarrow D_f = [1, 4]$ ✓

الف) $y = \frac{\sqrt{x^2 - 4x + 4}}{\sqrt{x - 10x + 14}} = \frac{\sqrt{(x-2)(x-1)}}{\sqrt{-9x+14}}$ $\frac{(x-2)(x-1)}{-9x+14} \geq 0 \Rightarrow \frac{(x-2)(x-1)}{14-9x} \geq 0$ $\Rightarrow D_f = (-\infty, 1] \cup [2, \frac{14}{9})$ ✓

ب) $\sqrt{\frac{x^2 - 4x + 4}{x^2 - 10x + 14}} = \sqrt{\frac{(x-2)(x-1)}{(x-1)(x-2)}}$ $\frac{(x-2)(x-1)}{(x-1)(x-2)} \geq 0 \Rightarrow \frac{1}{1} \geq 0 \Rightarrow D_f = (-\infty, 1) \cup (2, \infty)$ ✓

الف) $y = \sqrt{\frac{x^2 - 4x + 4}{x^2 - 10x + 14}} = \sqrt{\frac{(x-2)(x-1)}{(x-1)(x-2)}}$ $\frac{(x-2)(x-1)}{(x-1)(x-2)} \geq 0 \Rightarrow \frac{1}{1} \geq 0 \Rightarrow D_f = (-\infty, 1) \cup [2, \infty)$ ✓

ب) $y = \sqrt{\frac{x^2 - 4x + 4}{x^2 - 10x + 14}} = \sqrt{\frac{(x-2)(x-1)}{(x-1)(x-2)}}$ $\frac{(x-2)(x-1)}{(x-1)(x-2)} \geq 0 \Rightarrow \frac{1}{1} \geq 0 \Rightarrow D_f = (-\infty, \frac{14}{9}) \cup [\frac{14}{9}, \infty) \setminus \{1\}$ ✓