حل المسائل — پایه دهم

۱)

$A \times B = \{(1,2),(2,2),(3,2),(1,3),(2,3),(3,3)\} = 3 \times 2 = 6$

$B \times A = \{(2,2),(2,3),(3,2),(3,3)\} = 2 \times 2 = 4$

$\} \; 6 - 4 = 2$

۲) الف) $f(x) = \{(3,m^2),(2,1),(-3,m),(-2,m),(3,m+2),(m,4)\}$

$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0 \rightarrow m=2 \rightarrow (2,1)\,(2,4)$ غ ق ق

$\quad m=-1 \rightarrow \boxed{m=-1}$

ب) $g(x) = \{(3,m^2),(2,1),(m,4),(3,m+2),(-2,m),(-1,3)\}$

$m^2 = m+2 \rightarrow m=2 \rightarrow (2,4),(2,1)$ غ ق ق

$\quad m=-1 \rightarrow (-1,4),(-1,3)$ غ ق ق

هیچ مقدار m

۳-

$$f(x) = \begin{cases} x^2-1 & x \le 1 \quad (1) \\ ax+b & 1 \le x \le 2 \quad (2) \\ x^3 & x \ge 2 \quad (3) \end{cases}$$

$x=1 \quad (1),(2) \rightarrow a+b=0$

$x=2 \quad (2),(3) \rightarrow 2a+b=8$

$2a+b-(a+b)=8$

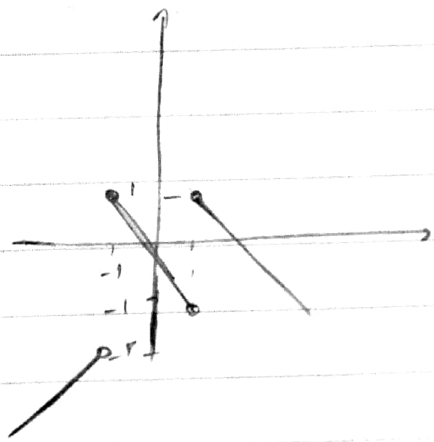
$a=8 \,,\; b=-8$

۴-

$3k-2 = 4-2k \rightarrow 5k = 6 \rightarrow k = \frac{6}{5}$

۵-

$$h(x) = \begin{cases} -x+2 & x>1 \\ -x & -1 \le x < 1 \\ x-1 & x < -1 \end{cases}$$



$D_{h(x)} = \mathbb{R} - \{1\}$

$R_y = (-\infty, 1]$

۶- تعریف روش

الف) $2y^3 - x^2 + 1 = 0 \xrightarrow{x_1 = x_2} 2y_1^3 + 1 = x_1^2$

$2y_2^3 + 1 = x_2^2 \quad \} \quad 2y_1^3 + 1 = 2y_2^3 + 1$

$2y_1^3 = 2y_2^3 \rightarrow y_1 = y_2$

رضا محمدی حل المسائل

۶- ب) $x^2+4y^2-2x-12y+\underset{1+9}{10}=0 \to x^2-2x+1+4y^2-12y+9=0$

$(x-1)^2+(2y-3)^2=0$ $\left|\begin{matrix}1\\ \frac{3}{2}\end{matrix}\right.$

$x-1=0$ $2y-3=0$
$x=1$ $y=\frac{3}{2}$

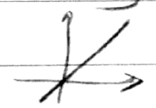
۷- الف) $x\sin y = y\sin x$ مثال نقض: $x=0$

$x=0 \Rightarrow y$ هر عددی می‌تواند

ب) $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=2 \xrightarrow{\text{به توان ۲}} \left(\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}\right)^2=4$

$\frac{x}{y}+\frac{y}{x}+2=4 \to \frac{x^2+y^2}{xy}=2 \to x^2-2xy+y^2=0$

$(x-y)^2=0 \to \boxed{x=y}$



۸- الف) $y=\sqrt{\frac{x-1}{x-3}}+\sqrt{\frac{2-x}{x}}$

$y_1=\sqrt{\frac{x-1}{x-3}} \Rightarrow$ 1, 3: + | − | + $D_{y_1}=(-\infty,1]\cup(3,+\infty)$

$y_2=\sqrt{\frac{2-x}{x}} \Rightarrow$ 0, 2: − | + | − $D_{y_2}=(0,2]$

$x=(0,1]$

ب) $\sqrt{x}+\sqrt{y-1}=\sqrt{3}$ $\sqrt{x}\geq 0$

$\sqrt{y-1}=\sqrt{3}-\sqrt{x}$

$\sqrt{3}-\sqrt{x}\geq 0 \to x\leq 3$

$0\leq x\leq 3 \to D_y=[0,3]$

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۹ـ

الف) $y = \dfrac{\sqrt{x^2-7x+6}}{\sqrt{x^2-10x+16}} = \dfrac{\sqrt{(x-6)(x-1)}}{\sqrt{(x-2)(x-8)}} \to D_y = (-\infty,1] \cup (8,+\infty)$

$\sqrt{(x-6)(x-1)} \Rightarrow$ $\overset{1}{+}\ -\ \overset{6}{+}$ $\to D_{y_1} = (-\infty,1] \cup [6,+\infty)$

$\sqrt{(x-2)(x-8)} \Rightarrow$ $\overset{2}{+}\ -\ \overset{8}{+}$ $\to D_{y_2} = (-\infty,2) \cup (8,+\infty)$

$\cap$

ب) $y = \sqrt{\dfrac{(x-6)(x-1)}{(x-2)(x-8)}}$

x		1		2		6		8	
x^2-7x+6	+	0	−		−	0	+		+
$x^2-10x+16$	+		+	0	−		−	0	+
	+	0	−	ت.ن	+	0	−	ت.ن	+

$D_y = (-\infty,1] \cup (2,6] \cup (8,+\infty)$

الف) $y = \dfrac{\sqrt{2x^2-5x+3}}{\sqrt{3x^2-7x+4}} = \dfrac{\sqrt{(x-1)(x-\frac{3}{2})}}{\sqrt{(x-1)(x-\frac{4}{3})}}$

$\overset{1}{+}\ -\ \overset{\frac{3}{2}}{+}$

$\overset{1}{+}\ \text{ت.ن}\ -\ \overset{\frac{4}{3}}{\text{ت.ن}}\ +$

$x'^2-5x'+6=0 \to (x'-2)(x'-3) \to x'=2 \;,\; x'=3 \quad \left\{ x=\frac{2}{2}=1 \;,\; x=\frac{3}{2}=\frac{3}{2} \right.$

$3x^2-7x+4=0 \to$ مجموع ضرایب 0 $\to x=1$

$x = \frac{c}{a} = \frac{4}{3}$

$D_1 = (-\infty,1] \cup [\frac{3}{2},+\infty)$

$D_2 = (-\infty,1) \cup (\frac{4}{3},+\infty)$ $\cap$ $D_1 = (-\infty,1) \cup [\frac{3}{2},+\infty)$

ب) $\sqrt{\dfrac{(x-1)(x-\frac{3}{2})}{(x-1)(x-\frac{4}{3})}}$

$\overset{1}{+\ \text{ت.ن}}\ +\ \overset{\frac{4}{3}}{\text{ت.ن}}\ -\ \overset{\frac{3}{2}}{+}$

$D = (-\infty,\frac{4}{3}) \cup [\frac{3}{2},+\infty) - \{1\}$

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