نام و نام خانوادگی: آیلین عظیمی　　تکلیف: ۱　　کلاس: انجوم یازدهم دختر

$A = \{1,2,3\}$

$B = \{2,3\} \Rightarrow B^2 = \{(2,2)(2,3)(3,2)(3,3)\}$

$\Rightarrow A \times B = \{(1,2)(1,3)(2,2)(2,3)(3,2)(3,3)\}$

$(A \times B) - B^2 = \boxed{\{(1,2)(1,3)\}}$ ۱

الف) $f(x) = \{(3,m^2)(2,1)(-3,m)(-2,m)(3,m+2)(m,4)\}$

$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \quad (m-2)(m+1) = 0$

$m = 2 \rightarrow$ غ ق ق $\rightarrow (2,1)(2,4)$ ✗

$\boxed{m = -1} \rightarrow (3,1)(2,1)(-3,-1)(-2,-1)(3,1)(-1,4)$ ✓

ب) $g(x) = \{(3,m^2)(2,1)(m,4)(3,m+2)(-2,m)(-1,3)\}$ ۲

$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \quad (m-2)(m+1)$

$m = 2$ غ ق ق $\rightarrow (2,1)(2,4)$ ✗

$m = -1$ غ ق ق $\rightarrow (-1,4)(-1,3)$ ✗

به ازای هیچ مقدار m این رابطه تابع نیست

$$f(x)\begin{cases} x^2 - 1 & x \le 1 \\ ax + b & 1 < x \le 2 \\ x^3 & x > 2 \end{cases}$$ ۳

$x = 1 \Rightarrow (1)^2 - 1 = a + b \rightarrow a + b = 0 \rightarrow a = -b$

$x = 2 \Rightarrow \underbrace{2a}_{-2b} + b = 8 \rightarrow -b = 8 \rightarrow b = 8,\ a = -8 \Rightarrow \boxed{ab = -64}$

$$f(x)\begin{cases} 2x - 3 & x \ge k \\ 4 - 3x & x \le k \end{cases}$$

$x = k \Rightarrow 2k - 3 = 4 - 3k$

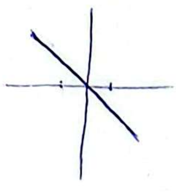
$5k = 7$

$\boxed{k = \frac{7}{5}}$ ۴

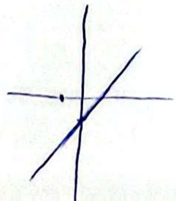
$$f(x)\begin{cases} -x + 2 & x \ge 1 \\ -x & -1 \le x < 1 \\ x - 1 & x < -1 \end{cases}$$

$\rightarrow y = -x + 2$:

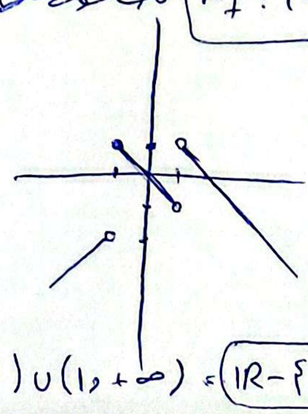
$\rightarrow y = -x$:



$y = x - 1$



$\Rightarrow$



$\boxed{R_f : (-\infty, 1]}$

$D_f : (-\infty, 1) \cup (1, +\infty) = \boxed{\mathbb{R} - \{1\}}$ ۵

الف) $2y^3+x^2+1=0 \to 2y_1^3=-x_1^2-1$, if $2y_2^3=-x_2^2-1 \to x_1=x_2 \to 2y_1^3=2y_2^3$

$y_1^3=y_2^3 \to \underline{y_1=y_2} \to$ تابع ✓ ۶

ب) $x^2+4y^2-2x-12y+10=0$

$x^2-2x+1+4y^2-12y+9=0 \to (x-1)^2+(2y-3)^2=0$, $x=1$, $y=\frac{3}{2}$ $\to \left(1, \frac{3}{2}\right)$ تک‌نقطه $\to$ تابع ✓

الف) $x\sin y=y\sin x \Rightarrow$ if $\sin x=0$ ($x=0$), $y\in\mathbb{R}$ $\Rightarrow$ ✗ تابع ۷

ب) $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=2 \xrightarrow{(\ )^2} \frac{x}{y}+\frac{y}{x}+2=4 \to \frac{x}{y}+\frac{y}{x}=2 \to \frac{x^2+y^2}{xy}=2 \to x^2+y^2-2xy=0$

$(x-y)^2=0$

$x=y$ تابع ✓

الف) $y=\sqrt{\frac{x-1}{x-3}}+\sqrt{\frac{2-x}{x}} \Rightarrow \cap \begin{cases} \frac{x-1}{x-3}\geq 0 & \begin{array}{c} 1 \quad 3 \\ + \ \ - \ \ + \end{array} \to (-\infty,1]\cup(3,+\infty) \\ \frac{2-x}{x}\geq 0 & \begin{array}{c} 0 \quad 2 \\ - \ \ + \ \ - \end{array} \to (0,2] \end{cases}$

$\boxed{D_f:(0,1]}$ ۸

ب) $y=\sqrt{x}+\sqrt{y-1}=\sqrt{3} \to \sqrt{y-1}=\sqrt{3}-\sqrt{x} \Rightarrow \sqrt{3}-\sqrt{x}\geq 0 \to \sqrt{x}\leq\sqrt{3} \to x\leq 3$, $x\geq 0$ $\to \boxed{D_f:[0,3]}$

الف) $y=\frac{\sqrt{x^2-7x+6}}{\sqrt{x-10x+16}}=\frac{\sqrt{(x-1)(x-6)}}{\sqrt{-9x+16}} \ \cap \begin{cases} (x-1)(x-6)\geq 0 & \begin{array}{c} 1 \quad 6 \\ + \ \ - \ \ + \end{array} \to (-\infty,1]\cup[6,+\infty) \\ 16-9x>0 \end{cases}$

$9x<16 \to x<\frac{16}{9}$ $\Rightarrow \boxed{D_f:(-\infty,1]}$ ۹

ب) $y=\sqrt{\frac{x^2-7x+6}{x^2-10x+16}} \to \frac{(x-1)(x-6)}{(x-2)(x-8)}\geq 0$ $\begin{array}{c} 1 \quad 2 \quad 6 \quad 8 \\ + \ \ - \ \ + \ \ - \ \ + \end{array} \to \boxed{D_f:(-\infty,1]\cup(2,6]\cup(8,+\infty)}$

الف) $y=\frac{\sqrt{2x^2-5x+3}}{\sqrt{3x^2-7x+4}} \to \cap \begin{cases} 2x^2-5x+3\geq 0 \to x^2-5x+6\geq 0 \\ (x-\frac{3}{2})(2x-3)\geq 0 \to (x-1)(2x-3)\geq 0 \\ 3x^2-7x+4>0 \\ \hookrightarrow x^2-7x+12>0 \\ (x-\frac{4}{3})(3x-4)>0 \to (x-1)(3x-4)>0 \end{cases}$

$\begin{array}{c} 1 \quad \frac{3}{2} \\ + \ \ - \ \ + \end{array}$ $(-\infty,1]\cup[\frac{3}{2},+\infty)$ ۱۰

$\begin{array}{c} 1 \quad \frac{4}{3} \\ + \ \ - \ \ + \end{array} \to (-\infty,1)\cup(\frac{4}{3},+\infty)$

$1 \quad \frac{4}{3} \quad \frac{3}{2}$

$\to \boxed{D_f:(-\infty,1)\cup[\frac{3}{2},+\infty)}$

ب) $y=\sqrt{\frac{2x^2-5x+3}{3x^2-7x+4}} \to \frac{(x-1)(2x-3)}{(x-1)(3x-4)}\geq 0$ $\begin{array}{c} 1^* \quad \frac{4}{3} \quad \frac{3}{2} \\ + \ \ + \ \ - \ \ + \end{array}$ $\boxed{D_f:(-\infty,1)\cup(1,\frac{4}{3})\cup[\frac{3}{2},+\infty)}$