

الف) $xy^2 + x^2 + 1 = 0 \rightarrow xy_1^2 = -x_1^2 - 1 \rightarrow$ if $xy_1^2 = -x_1^2 - 1$ $\rightarrow x_1 = x_2 \rightarrow xy_1^2 = xy_2^2$ $\rightarrow y_1 = y_2 \rightarrow$ تابع $\checkmark$

ب) $x^2 + 4y^2 - 2x - 12y + 1 = 0$
 $x^2 - 2x + 1 + 4y^2 - 12y + 9 = 0 \rightarrow (x-1)^2 + (2y-3)^2 = 0$ $\checkmark$ $\rightarrow$ نقطة $\checkmark$ $\rightarrow$ تابع $\checkmark$

الف) $x \sin y = y \sin x \rightarrow$ if $\sin x = 0$ $\rightarrow x = 0$ $\Rightarrow$ تابع $\checkmark$

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{(\cdot)}$ $\frac{x}{y} + \frac{y}{x} + 2 = 4 \rightarrow \frac{x}{y} + \frac{y}{x} = 2 \rightarrow \frac{x^2 + y^2 - 2xy}{xy} = 0$ $\rightarrow (x-y)^2 = 0$ $\rightarrow x = y$ $\checkmark$ $\rightarrow$ تابع $\checkmark$

الف) $y = \sqrt{\frac{x-1}{x-4}} + \sqrt{\frac{x-4}{x}}$ $\Rightarrow$ $\frac{x-1}{x-4} \geq 0 \rightarrow (-\infty, 1] \cup (4, +\infty)$ $\checkmark$ $\rightarrow$ $\frac{x-4}{x} \geq 0 \rightarrow [0, 4]$ $\checkmark$ $\rightarrow D_f = (0, 1]$

ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{y} \rightarrow \sqrt{y-1} = \sqrt{y} - \sqrt{x} \Rightarrow \sqrt{y-1} \geq 0 \rightarrow \sqrt{y} \geq \sqrt{x} \rightarrow x \leq y$ $\checkmark$ $\rightarrow D_f = [0, 1]$

الف) $y = \frac{\sqrt{x^2 - 5x + 4}}{\sqrt{x - 1.5x + 14}} = \frac{\sqrt{(x-1)(x-4)}}{\sqrt{-9x + 14}}$ $\cap \left\{ \begin{array}{l} (x-1)(x-4) \geq 0 \rightarrow (-\infty, 1] \cup [4, +\infty) \\ -9x + 14 > 0 \rightarrow 9x < 14 \rightarrow x < \frac{14}{9} \end{array} \right.$ $\checkmark$ $\rightarrow D_f = (-\infty, 1]$

ب) $y = \sqrt{\frac{x^2 - 5x + 4}{x^2 - 1.5x + 14}} \rightarrow \frac{(x-1)(x-4)}{(x-2)(x-1)} \geq 0 \rightarrow \frac{1}{x-2} + \frac{2}{x-4} + \frac{4}{x-1} + \frac{1}{x-2} \rightarrow D_f = (-\infty, 1] \cup (2, 4] \cup (1, +\infty)$ $\checkmark$

الف) $y = \frac{\sqrt{x^2 - 8x + 3}}{\sqrt{x^2 - 5x + 2}} \rightarrow \cap \left\{ \begin{array}{l} x^2 - 8x + 3 \geq 0 \rightarrow (x-4)(x-1) \geq 0 \rightarrow (-\infty, 1] \cup [4, +\infty) \\ x^2 - 5x + 2 > 0 \rightarrow (x-2)(x-1) > 0 \rightarrow (-\infty, 1) \cup (2, +\infty) \end{array} \right.$ $\checkmark$ $\rightarrow D_f = (-\infty, 1) \cup [2, +\infty)$

ب) $y = \sqrt{\frac{x^2 - 8x + 3}{x^2 - 5x + 2}} \rightarrow \frac{(x-1)(x-4)}{(x-1)(x-2)} \geq 0 \rightarrow \frac{1}{x-2} + \frac{2}{x-4} + \frac{4}{x-1} + \frac{1}{x-2} \rightarrow D_f = (-\infty, 1) \cup (2, \frac{3}{2}) \cup [\frac{3}{2}, +\infty)$ $\checkmark$