

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x < 1 \end{cases}$$

$$rf(a) = f(-r) + a$$

$$\hookrightarrow r(a+1)r = 3a - r + a$$

$$1ra + 1r = 4a - r \Rightarrow 1.a = -1r$$

$$a = -\frac{1}{r} \Rightarrow \left(-\frac{1}{a}\right)$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(r\sqrt{x}) + f(\frac{r}{\sqrt{x}}) = f(x) + f(\frac{1}{x}) \quad ①$$

$$\textcircled{1} \Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} + 2 + \frac{1}{\sqrt{x}} + \sqrt{x} + 2 \Rightarrow r(\sqrt{x} + \frac{1}{\sqrt{x}}) + 4 \Rightarrow (\sqrt{x} + \frac{1}{\sqrt{x}})^2 = x + \frac{1}{x} + 2$$

$$\Rightarrow r - \frac{1}{r} + r + \frac{1}{r} + 2 = 4 \Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{4} \Rightarrow \boxed{\sqrt{4} + 2}$$

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$$\begin{aligned} rf(x) - rf(-x) &= 4x^2 - x & rf(x) - rf(-x) &= 4x^2 - 2x \\ \Rightarrow rf(-x) - rf(x) &= 4x^2 - x & \Rightarrow -4f(x) &= 4x^2 - x \\ \Rightarrow rf(x) - rf(-x) &= 4x^2 - x & \Rightarrow 4f(x) &= 4x^2 - x \\ f(x) &= -x^2 - \frac{x}{4} \end{aligned}$$

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$$(x+r)f(x) - xf(x+r) = 4x^2 - mx + r - 1$$

$$\xrightarrow{x=0} rf(0) = r - 1 \Rightarrow f(0) = \frac{r-1}{r}$$

$$\xrightarrow{x=r} 4f(0) = 1 + r + r - 1 \Rightarrow 4f(0) = 1 + r + rf(0) \Rightarrow rf(0) = 1 + r \Rightarrow \boxed{rf(0) = 1 + r}$$

$$rm - 1 = 1 + r \Rightarrow rm = 2 \Rightarrow m = \frac{2}{r} \Rightarrow f(0) = \frac{\frac{2}{r} - 1}{r} = \frac{\frac{2}{r} - 1}{r} = \frac{2}{r^2} = f(0)$$

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$$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x} \Rightarrow ax + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \Rightarrow ax^2 + bx + a + b = 3x - 12 + 3$$

$$\begin{aligned} ax^2 &= 3x^2 \Rightarrow a = 3 \\ 3bx &= -12x \Rightarrow b = -4 \end{aligned} \Rightarrow 3x - 4 = f(x) \xrightarrow{x=-1} f(-1) = -1 - 4 \Rightarrow f(-1) = -5$$

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$$\left\{ \begin{array}{l} f(x) = (x-1)\sqrt{1-x} \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1 = Df \quad x = -2 \rightarrow -3\sqrt{3} \\ g(x) = -\sqrt{1-x}^3 \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1 = Dg \Rightarrow Df = Dg = (-\infty, 1] \\ x = -2 \rightarrow -\sqrt{27} = -3\sqrt{3} \end{array} \right.$$

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در تابع صدای اند

$$\left\{ \begin{array}{l} f(x) = x\sqrt{x} \Rightarrow x \geq 0 = Df \quad x = 4 \rightarrow 4\sqrt{4} = 8 \\ g(x) = \sqrt{x^3} \Rightarrow x \geq 0 = Dg \Rightarrow Df = Dg = [0, \infty) \\ x = 4 \rightarrow \sqrt{4^3} = \sqrt{64} = 8 \end{array} \right.$$

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در تابع صدای اند

$$\left\{ \begin{array}{l} f(x) = [\sqrt{x} - \sqrt{-x}] \Rightarrow x \geq 0 = Df \rightarrow x = 4 \rightarrow [\sqrt{4} - \sqrt{-4}] \rightarrow [2 - 2] = 0 \\ g(x) = \sqrt{x-1} \quad \begin{cases} \textcircled{1} \checkmark \rightarrow x=1 \rightarrow g(x) = \sqrt{1-1} \rightarrow g(x) = 0 \\ \textcircled{2} \times \rightarrow x=-1 \rightarrow g(x) = \sqrt{-1-1} \rightarrow g(x) = \sqrt{-2} \text{ تن} \\ \textcircled{3} \checkmark \rightarrow x=0 \rightarrow g(x) = 0 \end{cases} \Rightarrow Df = Dg \\ x-1 \geq 0 \quad Dg = x \geq 0 \\ |x| \leq x \end{array} \right.$$

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در تابع صدای اند

$$f = \{(1,4), (2,4), (3,1), (4,0)\} \quad g = \{(-1,1), (1,1), (2,4), (3,5), (4,2), (\sqrt{5}, 1)\}$$

الف) $\left(\frac{f \circ g}{f-g}\right)(1) : \frac{f(1)}{f(1)-g(1)} = \frac{4}{4-1} = 4$

ب) $\left(\frac{f \circ g}{f-g}\right)(2) : \frac{f(2)}{f(2)-g(2)} = \frac{4}{4-4} = \frac{4}{0}$ تعریف نشده

تاریخ زده شده

ج) $f(0)g(-1) : 0 \times 1 = 0$

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$$f(x) = \{(2,1), (-1,2), (0,5), (-2,2), (3,-1)\}$$

الف) $g(x) = f(x-1) \Rightarrow \begin{cases} x-1=2 \rightarrow x=3 \\ x-1=-1 \rightarrow x=0 \\ x-1=0 \rightarrow x=1 \\ x-1=-2 \rightarrow x=-1 \\ x-1=3 \rightarrow x=4 \end{cases} \Rightarrow g(x) = \{(3,1), (0,2), (1,5), (-1,2), (4,-1)\}$

ب) $h(x) = f(|x|) \Rightarrow \begin{cases} |x|=0 \rightarrow x=0 \\ |x|=2 \rightarrow x=2 \\ |x|=3 \end{cases} \Rightarrow h(x) = \{(0,5), (2,1), (3,1), (-3,1), (-2,1)\}$

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