

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x < 1 \end{cases}$$

$$2f(a) = f(-2) + a$$

$$\hookrightarrow 2(a+1) \cdot 4 = 3a - 4 + a$$

$$12a + 8 = 4a - 4 \Rightarrow 1 \cdot a = -12$$

$$a = -12 \Rightarrow \boxed{-12}$$

۱

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2\sqrt{x}) + f(\frac{1}{2\sqrt{x}}) = f(x) + f(\frac{1}{x}) \quad ①$$

$$\stackrel{①}{\Rightarrow} \sqrt{2x} + \frac{1}{\sqrt{2x}} + 2 + \frac{1}{\sqrt{2x}} + \sqrt{x} + 2 \Rightarrow 2(\sqrt{x} + \frac{1}{\sqrt{x}}) + 4 \Rightarrow (\sqrt{x} + \frac{1}{\sqrt{x}})^2 = x + \frac{1}{x} + 2$$

$$\Rightarrow x - \sqrt{x} + 2 + \sqrt{x} + 2 = 4 \Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{4} \Rightarrow \boxed{\sqrt{4} + 2}$$

۲

$$\begin{aligned} x^2 \quad 2f(x) - 4f(-x) &= 4x^2 - n \\ x = -x \quad \Rightarrow \quad 2f(-x) - 4f(x) &= 4x^2 - n \\ \Rightarrow \quad 2f(x) - 4f(-x) &= 4x^2 - n \\ \Rightarrow \quad -2f(x) &= 4x^2 - n \\ f(x) &= -2x^2 + \frac{n}{2} \end{aligned}$$

۳

$$(x+2)f(x) - 4xf(x+2) = 4x^2 - mx + 2m - 1$$

$$\stackrel{x=0}{\Rightarrow} 2f(0) = 2m - 1 \Rightarrow f(0) = \frac{2m-1}{2}$$

$$\stackrel{x=-2}{\Rightarrow} 4f(0) = 16 + 2m + 2m - 1 \Rightarrow 4f(0) = 16 + 4m + 2f(0) \Rightarrow 2f(0) = 16 + 4m \Rightarrow f(0) = 8 + 2m$$

$$2m - 1 = 16 + 4m \Rightarrow 2m = 17 \Rightarrow m = \frac{17}{2} \Rightarrow f(0) = \frac{17}{2} - 1 = \frac{15}{2} = \frac{15}{2} = f(0)$$

۴

$$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x} \Rightarrow ax^2 + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \Rightarrow ax^2 + bx + a + b = 3x - 12 + 3$$

$$\begin{aligned} ax^2 &= 3x^2 \Rightarrow a = 3 \\ 3bx &= -12x \Rightarrow b = -4 \\ \Rightarrow 3x - 4 &= f(x) \xrightarrow{x=-1} f(-1) = 3(-1) - 4 = -7 \end{aligned}$$

۵

$$\left\{ \begin{array}{l} f(x) = (x-1)\sqrt{1-x} \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1 = Df \quad x=0 \rightarrow -2\sqrt{1} \\ g(x) = -\sqrt{1-x} \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1 = Dg \Rightarrow Df = Dg = (-\infty, 1] \\ x=0 \rightarrow -\sqrt{1-0} = -1 \end{array} \right.$$

۶

در تابع مساوی اند.

$$\left\{ \begin{array}{l} f(x) = x\sqrt{x} \Rightarrow x \geq 0 = Df \quad x=4 \rightarrow 4\sqrt{4} = 8 \\ g(x) = \sqrt{x^3} \Rightarrow x \geq 0 = Dg \\ \Rightarrow Df = Dg = [0, \infty) \\ x=4 \rightarrow \sqrt{4^3} = \sqrt{64} = 8 \end{array} \right.$$

۷

در تابع مساوی اند.

$$\left\{ \begin{array}{l} f(x) = [\sqrt{x} - \sqrt{-x}] \Rightarrow x \geq 0 = Df \rightarrow x=1 \rightarrow [\sqrt{1} - \sqrt{-1}] \rightarrow [1 - i] \Rightarrow [1, i-1] = 0 \\ \Rightarrow Df = Dg \\ g(x) = \sqrt{x-1} \quad \begin{cases} \textcircled{1} \checkmark \rightarrow x=1 \rightarrow g(x) = \sqrt{1-1} \rightarrow g(x)=0 \\ \textcircled{2} \times \rightarrow x=-1 \rightarrow g(x) = \sqrt{-1-1} \rightarrow g(x) = \sqrt{-2} \text{ تن} \\ \textcircled{3} \checkmark \rightarrow x=0 \rightarrow g(x)=0 \end{cases} \quad Dg = x \geq 0 \\ x-1 \geq 0 \\ |x| \leq x \end{array} \right.$$

۸

در تابع مساوی اند.

$$f = \{(1,4), (2,4), (3,1), (4,0)\} \quad g = \{(-1,1), (1,1), (2,4), (3,5), (4,2), (\sqrt{5}, 1)\}$$

الف) $\left(\frac{f \cdot g}{f-g}\right)(1) : \frac{4 \cdot 1}{4-1} = \frac{4}{3}$

ب) $\left(\frac{f \cdot g}{f-g}\right)(2) : \frac{4 \cdot 4}{4-4} = \frac{16}{0}$ تعریف نشده

ج) $f(0)g(-1) : 0 \cdot 1 = 0$

۹

از این چهار ستیغی

$$f(x) = \{(2,1), (-1,2), (0,5), (-2,2), (3,-1)\}$$

الف) $g(x) = f(x-1) \Rightarrow \begin{cases} x-1=2 \rightarrow x=3 \\ x-1=-1 \rightarrow x=0 \\ x-1=0 \rightarrow x=1 \\ x-1=-2 \rightarrow x=-1 \\ x-1=3 \rightarrow x=4 \end{cases} \Rightarrow g(x) = \{(3,1), (0,2), (1,5), (-1,2), (4,-1)\}$

ب) $h(x) = f(|x|) \Rightarrow \begin{cases} |x|=0 \rightarrow x=0 \\ |x|=2 \rightarrow x=2 \\ |x|=3 \end{cases} \Rightarrow g(x) = \{(0,5), (2,1), (3,1), (-3,1)\}$

۱۰