① $\cot\frac{\frac{2}{3}\pi}{\frac{\pi}{3}} + \cot\frac{1}{4}\pi + \cot 30^\circ = \sqrt{(\sqrt{3})^2+1} + \sqrt{4} = 2$

② الف) $f(g(\frac{\pi}{2})) \Rightarrow g(\frac{\pi}{2}) = \frac{1}{2}$

$f(\frac{1}{\frac{1}{x}}) = \sqrt{\frac{2\frac{1}{x}-1}{\frac{1}{x^2}}} \longrightarrow f(\frac{1}{2}) = \sqrt{\frac{\frac{2}{2}-1}{\frac{1}{(\frac{1}{2})^2}}} = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

ب) $f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} = -2-\sqrt{2}$

$f(-2-\sqrt{2}) = [-2-\sqrt{2}] \longrightarrow f(-2-\sqrt{2}) = [-4]$

③ $g(F(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \rightarrow g(\frac{\sqrt{2}}{2}) = \sqrt{\frac{2}{2}} \times \sqrt{1-(\frac{\sqrt{2}}{2})^2} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

④ الف) $f(g(2,4,6,8))$

$f(g(4)) = f(8) = 12$

$f(g(6)) = f(10) = 12$

$f(g(2)) = f(4) = 5$

$f(g(4)) = f(6) = 5$

ب) $g(F(4,6,8,10)) = \varnothing$

ج) $F(f(x)) = \varnothing \rightarrow f(12) = \varnothing$, $f(5) = \varnothing$

(f: $2, 4, 6, 10$)

د) $g(g(6,4,8,2))$

$g(g(6)) = 8 \longrightarrow g(8) = 10$

$g(g(8)) = 10 \longrightarrow g(10) = \varnothing$

$g(g(4)) = 6 \longrightarrow g(6) = 8$

$g(g(2)) = 4 \longrightarrow g(4) = 6$

(۳)

$(\varepsilon,۲)\in f\circ g \rightarrow f\circ g(\varepsilon)=۲$

$f\circ g(x)=f(g(x))$

$f(g(\varepsilon))=۲$

$F(۲)=۲$

$g(\varepsilon)=۳ \rightarrow a=\varepsilon \Rightarrow (a,۲)=(\varepsilon,۲)$

$(\varepsilon,۱)\in g\circ f$

$g\circ f(\varepsilon)=۱$

$g(f(\varepsilon))=۱ \rightarrow F(\varepsilon)=۵ \quad (۳,۱),(b,۱) \rightarrow b=۵$

$g(۵)=۱$

$F(f(x))=۴x+۳$

$g(۲x+۲)=۲x-۲ \rightarrow f(x)=ax+b$ (۴)

$F(f(x))=۴x+۳$

$f(f(x))=a(ax+b)+b=a^۲x+ab+b$

$۴x+۳$

$a^۲=۴ \rightarrow a=۲ \rightarrow ۲b+b=۳ \Rightarrow ۳b=۳ \rightarrow b=۱ \rightarrow f(x)=۲x+۱$

$a=-۲ \rightarrow (-۲)b+b=۳ \Rightarrow -b=۳ \rightarrow b=-۳ \rightarrow f(x)=-۲x-۳$

$a\,b+b=۳ \rightarrow a=-۲$

$g(۲x+۲)=۲x-۲ \rightarrow t=۲x+۲ \rightarrow x=\frac{t-۲}{۲}$ $\quad g(t)=۲(\frac{t-۲}{۲})-۲=\frac{۲t-۴}{۲}-۲=\frac{۲t-۸}{۲}=g(f)$

$g(f(x))=۲f(x)+۱ \rightarrow g(f(x))=g(۲x+۱)=\frac{۲(۲x+۱)-۸}{۲}=\frac{۴x+۲-۸}{۲}=\frac{۴x-۶}{۲}=۲x-۳$ (۸)

$f(x)=-۲x-۳ \rightarrow g(f(x))=g(-۲x-۳)=\frac{۲(-۲x-۳)-۸}{۲}=\frac{-۴x-۶-۸}{۲}=-۲x-۷$ (۱۰)

$g(f(-۱))=-۸$ (۱۱)

$f(x)=\sqrt{x+|x|} \Rightarrow |x|=\begin{cases} x & x\geq ۰ \\ -x & x<۰ \end{cases}$ (۷)

$f(x)=\begin{cases} \sqrt{x+x}=\sqrt{۲x} \rightarrow ۲x\geq ۰ \rightarrow x\geq ۰ \\ \sqrt{x-x}=\sqrt{۰}=۰ & x<۰ \end{cases}$

$D_f=\mathbb{R}$

$g(x)=\frac{۱}{x^۲-۴x} \rightarrow x(x-۴)$

$f(x)\in D_g \rightarrow f(x)\neq ۰,۴$

$F(x) \dashrightarrow f(x)=۰ \rightarrow x=۰$

$f(۰)=\sqrt{۰}=۰ \rightarrow x=۰$

$f(x)=۴ \rightarrow \sqrt{۲x}=۴ \Rightarrow ۲x=۱۶ \Rightarrow x=۸$

$D_{g\circ f}=(۰,+\infty)-\{۸\}$

$(f+g)\circ f=\sqrt{۱-(f(x))^۲}+\sqrt{f(x)} \rightarrow f(x)=\sqrt{۱-x^۲} \rightarrow (\sqrt{۱-x^۲})^۲=۱-x^۲$ (۸)

① $۱-x^۲\geq ۰ \rightarrow -۱\leq x\leq ۱$

② $g(f(x))=\sqrt{f(x)} \rightarrow f(x)\geq ۰ \rightarrow f(x)=\sqrt{۱-x^۲}\geq ۰$

$D=[-۱,۱]$

$\frac{۲x+۱}{x-۲}=t \rightarrow tx-۲t=۲x+۱ \rightarrow tx-۲x=۲t+۱$ (۹)

$x(t-۲)=۲t+۱$

$x=\frac{۲t+۱}{t-۲}$

$f(t)=\frac{۸t+۴}{t-۲}+\frac{۵t-۱۵}{t-۲}$

$\frac{۱۳t-۱۱}{t-۲}$

$x+\frac{۱}{x}=t \rightarrow (x+\frac{۱}{x})^۲=x^۲+۲x\frac{۱}{x}+\frac{۱}{x^۲}$ (۱۰)

$x^۲+\frac{۱}{x^۲}=(x+\frac{۱}{x})^۲-۲(x+\frac{۱}{x}) \rightarrow x+\frac{۱}{x}=t \rightarrow f(t)=t^۲-۲t$

$f(x)=x^۲-۲x$

$g(1) = 0$

$g(2\sqrt{2}) = 0$

$f(x) = x\sqrt{x}$

$g \circ f(x) > g[f(x)] < 0$

$x\sqrt{x} = 1$

الف) $x^{\frac{3}{2}} = 1 \longrightarrow x = 1$

ب) $x^{\frac{3}{2}} = 2\sqrt{2} = 2 \times 2^{\frac{1}{2}} = 2^{\frac{3}{2}} \Rightarrow x^{\frac{3}{2}} = 2^{\frac{3}{2}} \Rightarrow x = 2$

$x_2 = 2 \quad x_1 = 1$

$|x_2 - x_1| = 2 - 1 = 1$

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