

$$f(x) = \begin{cases} C_0 + \frac{12x}{x} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$$

$$f \circ f\left(\frac{1}{4}\right) = f\left(f\left(\frac{1}{4}\right)\right) = f(\sqrt{3}) = 2$$

$$f\left(\frac{1}{4}\right) = C_0 + \frac{12 \times \frac{1}{4}}{\frac{1}{4}} = \frac{C_0 + 12}{\frac{1}{4}} = C_0 + 12 = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = 2$$

$$2) f\left(\frac{1}{x}\right) = \frac{\sqrt{x^2 - 1}}{x^2} \quad \frac{1}{x} = t, \quad f(t) = \frac{\sqrt{t^2 - 1}}{\left(\frac{1}{t}\right)^2} = \sqrt{t^2 \left(\frac{1}{t} - 1\right)} = \sqrt{1 - t}$$

$$\sqrt{t(1-t)} \rightarrow g\left(\frac{1}{x}\right) = 2 \cos^2\left(\frac{x}{4}\right) = 2 \times \left(\frac{1}{x}\right)^2 = \frac{1}{x} \quad f \circ g\left(\frac{1}{x}\right) = f\left(g\left(\frac{1}{x}\right)\right) = \frac{1}{\frac{1}{x}} = x$$

$$b) g(\sqrt{x}) = \frac{(\sqrt{x})(1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})} = \frac{\sqrt{x}+x}{-1} = -\sqrt{x}-x$$

$$f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x} \left(\frac{1}{x} - 1\right)} = \sqrt{\frac{1-x}{x}} = \frac{\sqrt{1-x}}{\sqrt{x}} \quad f(g(\sqrt{x})) = -\sqrt{x}$$

$$f(-\sqrt{x}-x) = [-\sqrt{x}-x] = [-x-x] = -2x$$

$$f\left(\frac{a}{x}\right) = \sin \frac{a}{x} = \frac{\sqrt{x}}{x} \quad g\left(\frac{\sqrt{x}}{x}\right) = \frac{\sqrt{x}}{x} \sqrt{1 - \left(\frac{\sqrt{x}}{x}\right)^2} = \frac{\sqrt{x}}{x} \times \sqrt{\frac{x-x}{x}} = \frac{\sqrt{x}}{x} \times \frac{1}{\sqrt{x}} = \frac{1}{x}$$

$$f(x) = \{(\varepsilon, \omega), (y, \omega), (\wedge, 1^2), (1, 1^2)\}$$

$$g(x) = \{(\varepsilon, y), (y, \varepsilon), (y, \wedge), (\wedge, 1, 2)\}$$

$$f \circ g(x) = x = \varepsilon \xrightarrow{f} y \xrightarrow{g} \omega \quad y \xrightarrow{f} \omega \xrightarrow{g} \omega \quad \omega \xrightarrow{f} \omega \xrightarrow{g} \omega \quad \omega \xrightarrow{f} \omega \xrightarrow{g} \omega$$

$$y \rightarrow \wedge \rightarrow 1^2 = \{(\varepsilon, \omega), (y, \omega), (y, 1^2), (\wedge, 1^2)\}$$

$$g \circ f(x) = \emptyset \quad x = \varepsilon \xrightarrow{f} \omega \xrightarrow{g} x \quad \omega \xrightarrow{f} \omega \xrightarrow{g} x \quad \omega \xrightarrow{f} \omega \xrightarrow{g} x \quad \omega \xrightarrow{f} \omega \xrightarrow{g} x$$

$$y \rightarrow \omega \rightarrow x \quad \wedge \rightarrow 1^2 \rightarrow x \quad 1, 2 \rightarrow 1^2 \rightarrow x$$

$$f \circ f(x) = \emptyset \quad x = \varepsilon \xrightarrow{f} \omega \xrightarrow{f} x \quad \omega \xrightarrow{f} \omega \xrightarrow{f} x \quad \omega \xrightarrow{f} \omega \xrightarrow{f} x \quad \omega \xrightarrow{f} \omega \xrightarrow{f} x$$

$$y \rightarrow \omega \rightarrow x \quad \wedge \rightarrow 1^2 \rightarrow x \quad 1, 2 \rightarrow 1^2 \rightarrow x$$

$$g \circ g(x) = x = \varepsilon \xrightarrow{g} y \xrightarrow{g} \wedge \quad y \xrightarrow{g} y \xrightarrow{g} \wedge \quad \omega \xrightarrow{g} \omega \xrightarrow{g} \wedge \quad \omega \xrightarrow{g} \omega \xrightarrow{g} \wedge$$

$$y \rightarrow \varepsilon \rightarrow y \quad y \rightarrow \wedge \rightarrow 1, 2 \quad \wedge \rightarrow 1, 2 \rightarrow x$$

$$f = \{(2, 1), (3, 2), (\varepsilon, \omega), (1, v)\}$$

$$g = \{(b, 2), (3, 1), (a, 2), (b, \omega), (1, v)\}$$

$$f \circ g(\varepsilon) = 2 \Rightarrow x = \varepsilon \xrightarrow{g} 1 \xrightarrow{f} 2$$

$$a \rightarrow 3 \rightarrow 2 \checkmark \rightarrow a = \varepsilon \quad b \rightarrow 1 \rightarrow v \times$$

$$g \circ f(\varepsilon) = 1 \Rightarrow x = \varepsilon \xrightarrow{f} \omega \rightarrow 1$$

$$(b, 1) = (\omega, 1) \rightarrow b = \omega$$

$$(a, b) = (\varepsilon, \omega)$$

$$f(x) = ax + b \rightarrow f(f(x)) = f(ax + b) = a(ax + b) + b = a^2x + ab + b = \underline{ax + b}$$

$$\rightarrow a^2 = 1 \rightarrow a = \pm 1$$

$$a = 1 \rightarrow b = 1 \rightarrow b = 1 \rightarrow f(x) = x + 1$$

$$a = -1 \rightarrow -b = 1 \rightarrow b = -1 \rightarrow f(x) = -x - 1$$

$$g(x, t) = x - t$$

$$\rightarrow x - t = t \rightarrow x = \frac{t + t}{2}$$

$$\rightarrow g(t) = x - t = \frac{t + t}{2} - t = \frac{t - t}{2} = 0$$

$$\rightarrow g(x) = \frac{x - 1}{2}$$

$$g(f(-1)) = g(-1 + 1) = g(0) = \frac{0 - 1}{2} = -\frac{1}{2}$$

$$f(x) = \sqrt{x+1} \begin{cases} R^- = 0 \checkmark \\ R^+ = x \checkmark \end{cases}$$

$$D_f = R^+$$

$$g \circ f = \frac{1}{(f(x))^2 - f(x)}$$

$$\rightarrow f(x)(f(x) - 1) \neq 0 \begin{cases} f(x) \neq 0 \rightarrow R^- = 0 \times \\ f(x) \neq 1 \rightarrow \sqrt{x+1} = 1 \rightarrow x = 0 \times \end{cases}$$

$$D_{g \circ f} = (0, 1) \cup (1, +\infty)$$

$$(f+g) \circ f = f(f(x)) + g(f(x))$$

$$f(f(x)) = \sqrt{1 - (f(x))^2}$$

$$\sqrt{1 - (\sqrt{1-x^2})^2} = \sqrt{1 - (1-x^2)} = \sqrt{x^2} = |x|$$

$$D_g = x \neq 0 \quad D_{f+g} = -1 \leq x \leq 1$$

$$g(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{1-x^2}} = (1-x^2)^{\frac{1}{4}}$$

$$f(x) = \sqrt{1-x^2}$$

$$1 \geq x^2 \rightarrow -1 \leq x \leq 1$$

$$\sqrt{1-x^2} \geq 0$$

$$1-x^2 \geq 0 \rightarrow -1 \leq x \leq 1$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$f\left(\frac{x+1}{x-1}\right) = \frac{x+1}{x-1} \rightarrow \frac{x+1}{x-1} = t \rightarrow \frac{x+1}{t} = t-1 \rightarrow x = \frac{t(t-1)+1}{t-1}$$

$$f(t) = \frac{t(t-1)+1}{t-1} = \frac{t^2 - t + 1}{t-1} = \frac{t^2 - 1}{t-1} = \frac{(t-1)(t+1)}{t-1} = t+1 \rightarrow f(x) = \frac{x+1}{x-1}$$

$$b) f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow x + \frac{1}{x} = t \rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 \rightarrow x^2 + \frac{1}{x^2} + 2 = t^2 \rightarrow x^2 + \frac{1}{x^2} = t^2 - 2 \rightarrow f(t) = t^2 - 2 \rightarrow f(x) = x^2 - 2$$

$$g(f(x)) \rightarrow \sqrt{f(x)}, \text{ ل } f(x) \text{ أكبر من صفر}$$

$$f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$$

$$f(x) = \sqrt{x} \rightarrow x\sqrt{x} = \sqrt{x} \rightarrow x = 1$$

$$x-1=1$$