نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$f(f(\frac{\pi}{3})) = f(\sqrt{3}) = 2$

$f(\frac{\pi}{3}) = \frac{\cot \pi \times \frac{\pi}{3}}{\sin \frac{\pi}{3}} = \cot \frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) = \sqrt{3+1} = 2$

(۲)

۲

الف)

$f(g(\frac{\pi}{3}))$

$g(\frac{\pi}{3}) = 2\cos^2 \frac{\pi}{3} = \frac{1}{2}$

$f(\frac{1}{2}) = f(2) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

(۲)

ب)

$f(g(\sqrt{3}))$

$g(\sqrt{3}) = \frac{\sqrt{3}}{1-\sqrt{3}} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{3}+3}{-1} = -\sqrt{3}-3$

$f(-2-\sqrt{3}) = [\underbrace{-2-\sqrt{3}}_{-3<\cdots<-2}] = -3$

۳

$g(f(\frac{\pi}{4})) = g(\frac{\sqrt{2}}{2}) = \frac{1}{2}$

$f(\frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

(۲)

۴

الف)

$f(g(x)) = \{(4,3)(2,3)(9,12)(8,12)\}$

$4 \to 9 \to 3$
$2 \to 4 \to 3$
$9 \to 8 \to 12$
$8 \to 10 \to 12$

ب)

$g(f(x)) = ?$

$4 \to 3$ ✗
$9 \to 3$ ✗
$8 \to 12$ ✗
$10 \to 12$ ✗

ج)

$f(f(x)) = ?$

(۲)

$4 \to 3$ ✗
$9 \to 3$ ✗
$8 \to 12$ ✗
$10 \to 12$ ✗

$g(g(x)) = \{(4,8)(2,9)(9,10)\}$

$4 \to 9 \to 8$
$2 \to 4 \to 9$
$9 \to 8 \to 10$
$8 \to 10$ ✗

۵

$fog \begin{cases} 1 \to 2 \to 1 \\ 2 \to 1 \to 7 \\ a \to 3 \to 2 \\ b \to 1 \to 7 \end{cases}$

$\to (a,2) = (4,2) \to a = 4$

$\to (b,7) \neq (4,2)$ ✗

$(a,b) = (4,3)$

(۲)

$gof \begin{cases} 2 \to 1 \to 2 \\ 4 \to 2 \\ 4 \to 3 \to 1 \\ 1 \to 7 \end{cases}$

$\to (4,1) \to (b,1) = (3,1)$ $b = 3$

۶

$g(2x+2)=4x-2$ $\quad 2x+2=t$ $\quad x=\frac{t-2}{2}$ $\quad g(t)=4\left(\frac{t-2}{2}\right)-2=\frac{4t-12}{2}$ $\quad g(x)=\frac{4x-12}{2}$

$f(x)=ax+b \rightarrow f(ax+b)=4x+2 \rightarrow a(ax+b)+b$

$a^2x+ab+b=4x+2$ $\quad a^2=4 \begin{cases} a=-2 & f(x)=-2x+2 \\ a=2 & f(x)=2x+1 \end{cases}$ $f(-1)=-1$

(۲)

$g(f(x))=g(f(1))=g(-1)=\frac{-14}{2}=-7$

۷

$D_{gof}=\{x\in D_f, f(x)\in D_g\}$ R(۱)

$\sqrt{x+|x|}\neq 0 \quad x+|x|\neq 0$

$-x\neq|x| \quad x>0$ (۲)

$\sqrt{x+|x|}\neq 4 \quad x+|x|\neq 16 \quad x\neq 8$ (۳)

$D_{gof}=(۱)\cap(۲)\cap(۳)=(0,+\infty)-\{8\}$ (۲)

۸

$fof+fog$

$D_f: -1\le x\le 1 \quad \cup \quad D_g: x\ge 0 \rightarrow 0\le x\le 1$

$fof=\sqrt{1-1+x^2}=\sqrt{x^2}=|x| \quad D=[0,+\infty)$ (۱)

$(۱)\cap(۲)=[0,1]$ (۱)

$g\circ f=\sqrt{\sqrt{1-x^2}} \rightarrow 1-x^2\ge 0 \quad x^2\le 1$ (۲)

$D_{(f+g)of}=[-1,1]$

۹

$f(x+\frac{1}{x})=x^2+\frac{1}{x^2} \quad x+\frac{1}{x}=t$

$f(x+\frac{1}{x})=(x+\frac{1}{x})^2-2x-\frac{2}{x}=(x+\frac{1}{x})^2-2(x+\frac{1}{x})$

$f(t)=t^2-2t \rightarrow f(x)=x^2-2x$

(ب)

الف) $\frac{2x+1}{x-2}=t \quad x=\frac{2t+1}{t-2}$

$f\left(\underbrace{\frac{2x+1}{x-2}}_{t}\right)=4x+a$

$f(t)=4\left(\frac{2t+1}{t-2}\right)+a=\frac{8t+4+at-2a}{t-2}$

$=\frac{11-3t}{t-2}$ $\quad f(x)=\frac{11-3x}{x-2}$ (۲)

۱۰

$f(x)=x\sqrt{x}$ $\begin{cases} x\sqrt{x}=1 \rightarrow x=1 \\ x\sqrt{x}=2\sqrt{2} \rightarrow x=2 \end{cases}$ $\quad 2-1=1$ (۲)