

Subject

Date ١٤.٣ / ٧ / ١٣

ب نام خدا

(٢٠)

فاطمه متقی - ٩

$$f(f(\frac{\pi}{3})) \Rightarrow f(\frac{\pi}{3}) = \cot \frac{\pi}{3} \times \frac{\pi}{3} = \cot \frac{\pi}{3} = \sqrt{3}$$

(١) (٢)

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{3+1} = \sqrt{4} = 2 \Rightarrow f \circ f(\frac{\pi}{3}) = 2$$

$$f(g(\frac{\pi}{4})) \Rightarrow g(\frac{\pi}{4}) = 2 \cos^2 \frac{\pi}{4} = 2 \times (\frac{1}{2})^2 = \frac{1}{2}$$

(٢) الف (٢)

$$f(\frac{1}{2}) = \sqrt{\frac{2 \cos^2 - 1}{2}} = \sqrt{\frac{2 - 1}{2}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \Rightarrow f \circ g(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = -(\sqrt{2}+2) \approx -\sqrt{2}-2$$

(١) (٢)

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] \approx [-1, 2-2] = [-3, 2] = -4$$

$$g(f(\frac{\pi}{6})) \Rightarrow f(\frac{\pi}{6}) = \sin \frac{\pi}{6} = \frac{\sqrt{2}}{2}$$

(٣) (٢)

$$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \sqrt{1 - (\frac{\sqrt{2}}{2})^2} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$الف) f \circ g(a) = \{(4, 5), (2, 5), (9, 12), (1, 12)\}$$

(٣) (٢)

$$ب) g \circ f(a) = \emptyset$$

$$ج) f \circ f(a) = \emptyset$$

$$د) g \circ g(a) = \{(4, 1), (2, 4), (4, 1)\}$$

$$if a=4 \rightarrow g(4)=3 \rightarrow f(3)=2 \Rightarrow f \circ g = (4, 2) \Rightarrow \checkmark$$

(٥) (٢)

$$if b=4 \rightarrow g(4)=1 \rightarrow f(1)=\sqrt{2} \Rightarrow f \circ g = (4, \sqrt{2}) \Rightarrow X$$

$\rightarrow a=4$ ①

(٢)

$$g(f(4))=1 \rightarrow g(a)=1 \Rightarrow b=a \quad (٢) \quad \xrightarrow{\text{①, ①}} (a, b) = (4, 4)$$

$$f(4) = 5$$

$$f \circ f(x) = 2x + 3 \rightarrow \text{if } f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = 2x + 3 \quad (6)$$

$$\hookrightarrow a^2x + ab + b = 2x + 3 \rightarrow a^2 = 2 \rightarrow a = \pm \sqrt{2}$$

$$\left. \begin{array}{l} \text{if } a = \sqrt{2}, 2b + b = 3 \rightarrow b = 1 \Rightarrow f(x) = \sqrt{2}x + 1 \\ \text{if } a = -\sqrt{2}, -2b + b = 3 \rightarrow b = -3 \Rightarrow f(x) = -\sqrt{2}x - 3 \end{array} \right\} \rightarrow f(-1) = -1$$

$$g(2x + 3) = 3x - 2 \rightarrow \text{if } g(x) = cx + d \rightarrow g(2x + 3) = c(2x + 3) + d = 3x - 2$$

$$2c + 3c + d = 3x - 2 \rightarrow 5c = 3 \rightarrow c = \frac{3}{5}$$

$$d = -2 - \frac{9}{5} = -\frac{11}{5}$$

$$\left. \begin{array}{l} c = \frac{3}{5} \\ d = -\frac{11}{5} \end{array} \right\} \rightarrow g(x) = \frac{3}{5}x - \frac{11}{5}$$

$$g(f(-1)) = g(-1) = -\frac{3}{5} - \frac{11}{5} = -\frac{14}{5} = -\frac{14}{5}$$

$$g(f(x)) = \frac{1}{(\sqrt{x+|x|})^2 - \sqrt{x+|x|}} \rightarrow \frac{1}{|x+|x|| - \sqrt{x+|x|}} = \frac{1}{x+|x| - \sqrt{x+|x|}} \quad (7)$$

$$\hookrightarrow D_{g \circ f} = \mathbb{R} - \{x \mid x+|x| = 0\}$$

$$x+|x| - \sqrt{x+|x|} = 0 \quad t = \sqrt{x+|x|} \quad t^2 - \sqrt{x+|x|} = 0 \quad \left\{ \begin{array}{l} t = \sqrt{x+|x|} \Rightarrow x+|x| = t^2 \\ t = 0 \Rightarrow x \leq 0 \end{array} \right. \quad \begin{array}{l} x \leq 0 \Rightarrow x+|x| = 0 \\ x > 0 \Rightarrow x+|x| = 2x \end{array} \quad \begin{array}{l} 2x = t^2 \Rightarrow x = \frac{t^2}{2} \\ x = \frac{t^2}{2} \Rightarrow x+|x| = t^2 \end{array}$$

$$\hookrightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$$x \in D_f \Rightarrow x \in \mathbb{R}$$

$$(f+g) \circ f \Rightarrow f(f(x)) + g(f(x)) \Rightarrow D_{f \circ f} = 1 - x^2 > 0 \rightarrow x^2 < 1 \rightarrow [-1, 1] \quad (8)$$

$$\hookrightarrow f(f(x)) = \sqrt{1 - |1 - x^2|} \quad \begin{array}{l} \text{در بازه } [-1, 1] \\ \text{تقریب می‌نبرد} \end{array} \quad (1)$$

$$\hookrightarrow g(f(x)) = \sqrt{\sqrt{1 - x^2}} = \sqrt[4]{1 - x^2} \Rightarrow x^2 \leq 1 \quad \text{or} \quad x \leq 1 \quad \text{or} \quad x \geq -1 \rightarrow [-1, 1] \quad (2)$$

$$f(f(x)) + g(f(x)) \Rightarrow (1), (2) \Rightarrow D_{(f+g) \circ f} = [-1, 1]$$

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$$\text{الف) } f\left(\frac{y^2x+1}{x-y}\right) = f(x+y) \quad \frac{y^2x+1}{x-y} = t \rightarrow tx - yt = y^2x+1 \quad (9)$$

$$\rightarrow (y-t)x = -yt-1 \rightarrow x = \frac{-yt-1}{y-t} = \frac{yt+1}{t-y} \quad (3)$$

$$f\left(\frac{yt+1}{t-y}\right) = f\left(\frac{yt+1}{t-y}\right) + \omega \rightarrow f(x) = \frac{1^2x-1}{x-y} \Rightarrow (x \neq y)$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} \Rightarrow f(t) = t^3 - \frac{1}{t^3} \Rightarrow f(x) = x^3 - \frac{1}{x^3}$$
$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} - 3\left(x + \frac{1}{x}\right)$$

$$g(f(x)) = g(x\sqrt{x}) \xrightarrow{x=y} g(y\sqrt{y}) \quad (1)$$

$$g(x\sqrt{x}) \xrightarrow{x=1} g(1) \xrightarrow{y=1} \boxed{1} \quad (3)$$