نام و نام خانوادگی: جمال رجوامی .... پاسخنامه تشریحی تکلیف شماره ..۹... کلاس ....

۱

$$\begin{cases} \cot \frac{\pi x}{4} & x \le 1 \\ \sqrt{x^2+1} & x > 1 \end{cases}$$

$f(f(\frac{2}{3})) \rightarrow f(\frac{2}{3}) = \cot \frac{\pi(\frac{2}{3})}{4} = \cot \frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) \rightarrow \sqrt{3+1} = \sqrt{4} = 2$

۲

$fog(\frac{\pi}{3}) \rightarrow f(g(\frac{\pi}{3})) \rightarrow g(\frac{\pi}{3}) = 2 \times \cos^2(\frac{\pi}{3}) = 2 \times \frac{1}{4} = \frac{1}{2}$

$f(\frac{1}{x}) \rightarrow \frac{1}{x} = \frac{1}{2} \rightarrow x = 2 \rightarrow f(2) = \sqrt{\frac{3}{4}} \rightarrow \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2}$

$fog(\sqrt{2}) \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$

$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$

۳

$f(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} \Rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

۴

الف) $f(g(x)) \Rightarrow$ $4 \rightarrow 6 \rightarrow 5$, $2 \rightarrow 4 \rightarrow 5$, $6 \rightarrow 8 \rightarrow 12$, $8 \rightarrow 10 \rightarrow 12$ $\rightarrow \{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g(f(x)) \Rightarrow$ $4 \rightarrow 5 \rightarrow \times$, $6 \rightarrow 5 \rightarrow \times$, $8 \rightarrow 12 \rightarrow \times$, $10 \rightarrow 12 \rightarrow \times$ $\quad gof(x) = \varnothing$

ج) $f(f(x)) \rightarrow 4 \rightarrow 5 \rightarrow \times$, $6 \rightarrow 5 \rightarrow \times$, $8 \rightarrow 12 \rightarrow \times$, $10 \rightarrow 12 \rightarrow \times$ $\quad fof(x) = \varnothing$

د) $gog(x) \rightarrow$ $4 \rightarrow 6 \rightarrow 8$, $2 \rightarrow 4 \rightarrow 6$, $6 \rightarrow 8 \rightarrow 10$, $8 \rightarrow 10 \rightarrow \times$

$gog(x) = \{(4,8),(2,6),(6,1$

۵

$f(g(x)):\ 4 \rightarrow x \rightarrow 2 \qquad f(x) \rightarrow (3,2)$

$a \longrightarrow 3 \rightarrow 2 \qquad x = 3 \rightarrow a = 4$

$\rightarrow (4,5)$

$g(f(x)):\ 4 \rightarrow x \rightarrow 1 \rightarrow b = 5$

$(4,5) \qquad 5 \qquad (1,b)$

تابع خطی $\rightarrow f(x)=ax+b \rightarrow f(f(x)) = a(ax+b)+b = a^2x+ab+b = 4x+3$

$a^2=4 \rightarrow a=\pm 2 \rightarrow +2 = 4x+3b \ , \ b=1 \rightarrow f(x)=2x+1$

$-2 = 4x-b \ , \ b=-3 \rightarrow f(x)=-2x-3$ ۶

$g(\underbrace{2x+3}_{t})=3x-2 \rightarrow t=2x+3 \rightarrow x=\frac{t-3}{2} \rightarrow g(t)=3\left(\frac{t-3}{2}\right)-2=\frac{3t}{2}-\frac{9}{2}-2$

$g(-1)=\frac{-3(3)-9}{2}-2=-8$ $\quad -1 \leftarrow f(-1)=2(-1)+1$ $\quad -1 \leftarrow f(-1)=-2(-1)-3$ $\quad f(x)=\frac{3x-9}{2}-2$

$D_{gof}=\{x\in D_f : f(x)\in D_g\}$

$D_f \rightarrow x+|x|\geq 0 \rightarrow R$ $\quad \rightarrow \{R : \sqrt{x+|x|}\neq 4, 0\}$ ۷

$D_g \rightarrow x^2-4x\neq 0 \rightarrow x\neq 4 \rightarrow R-\{4,0\}$ $\quad \sqrt{x+|x|}\neq 4 \rightarrow x+|x|\neq 16 \rightarrow x\neq 8$

$D_{gof}=(0,+\infty)-\{8\} \leftarrow \sqrt{x+|x|}\neq 0 \rightarrow x\notin(-\infty,0]$

$D_{(f+g)of}=\{x\in D_f : f(x)\in D_{f+g}\} \rightarrow \{[-1,1] : f(x)\in\{0,1\}\}$

$D_{f(x)}=1-x^2\geq 0 \rightarrow D_f=[-1,1]$ $\quad \cap \quad \sqrt{1-x^2}=0 \rightarrow x=\pm 1$ ۸

$D_{f+g}=\{1,0\}$ $\quad f+g \rightarrow \{(1,1)\ (0,1)\}$ $\quad \sqrt{1-x^2}=1 \rightarrow x=0$

عددگذاری $\rightarrow f(x) \rightarrow \{(1,0)(0,1)\} \ , \ g(x) \rightarrow (1,1)\ (0,0)$ $\quad D_{(f+g)of}=[-1,1]$

الف) $\frac{3x+1}{x-2}=t \rightarrow 3x+1=tx-2t \rightarrow 3x-tx=-2t-1 \rightarrow x(3-t)=-2t-1 = -(2t+1)$

$f(t)=4\left(\frac{2t+1}{t-3}\right)+5 \rightarrow f(x)=4\left(\frac{2x+1}{x-3}\right)+5 \rightarrow \quad x=\frac{2t+1}{t-3}$ ۹

$f(x)=\frac{8x+4}{x-3}+5$

ب) $x+\frac{1}{x}=t$

$(x+\frac{1}{x})^3=x^3+\frac{1}{x^3}+\frac{3}{x}+3x \rightarrow x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3(\frac{1}{x}+x) \rightarrow f(t)=t^3-3t \rightarrow f(x)=x^3-3x$

$gof(x)=0 \rightarrow g(x)=0 \rightarrow f(x)=0 \rightarrow x\sqrt{x}=2\sqrt{2} \rightarrow x=2$

$\hookrightarrow x\sqrt{x}=1 \rightarrow x=1$ ۱۰