

$$\cot \frac{\pi \times \frac{1}{2}}{2} \Rightarrow \cot \frac{2\pi}{12} \Rightarrow \cot \frac{\pi}{6} \Rightarrow \cot 30^\circ = \sqrt{3}$$

$$\sqrt{3} > 1 \Rightarrow \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

الف) $f \circ g\left(\frac{\pi}{2}\right) = f(g(\pi)) = f\left(g\left(\frac{\pi}{2}\right)\right)$

$n = \frac{\pi}{2} \Rightarrow g\left(\frac{\pi}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$

$\rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

ب) $f \circ g(n) = f(g(n))$
 $= f(g(\sqrt{2})) =$
 $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}$
 $\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{1-2} = -(\sqrt{2}+2)$
 $= [g(\sqrt{2})] = [-2, -1, 2] = -4$

$$\frac{\pi}{2} \Rightarrow \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$g(n) = n \sqrt{1-n^2} \Rightarrow \frac{\sqrt{2}}{2} \sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \frac{1}{2} = \sqrt{\frac{1}{4} \times \frac{1}{2}} = \sqrt{\frac{1}{8}}$$

$$g \circ f\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

الف) $f \circ g(n) =$

۴	$\xrightarrow{g(n)}$	۶	$\xrightarrow{f(n)}$	۵
۲	$\xrightarrow{g(n)}$	۴	$\xrightarrow{f(n)}$	۵
۶	$\xrightarrow{g(n)}$	۸	$\xrightarrow{f(n)}$	۱۲
۸	$\xrightarrow{g(n)}$	۱۰	$\xrightarrow{f(n)}$	۱۲

$= \{(4, 5), (2, 5), (6, 12), (8, 12)\}$

ب) $g \circ f(n) = \emptyset$

ج) $f \circ f(n) = \emptyset$

د) $g \circ g(n) =$

۴	$\xrightarrow{g(n)}$	۶	$\xrightarrow{g(n)}$	۸
۲	$\xrightarrow{g(n)}$	۴	$\xrightarrow{g(n)}$	۶
۶	$\xrightarrow{g(n)}$	۸	$\xrightarrow{g(n)}$	۱۰
۸	$\xrightarrow{g(n)}$	۱۰		

$= \{(4, 8), (2, 6), (6, 10)\}$

$g \circ f$

۲	$\xrightarrow{f(n)}$	۱	$\xrightarrow{g(n)}$	۲
۴	$\xrightarrow{f(n)}$	۲		
۶	$\xrightarrow{f(n)}$	۵	$\xrightarrow{g(n)}$	۱۰
۸	$\xrightarrow{f(n)}$	۱۰		

$f \circ g$

۱	$\xrightarrow{g(n)}$	۲	$\xrightarrow{f(n)}$	۱
۲	$\xrightarrow{g(n)}$	۱	$\xrightarrow{f(n)}$	۵
۵	$\xrightarrow{g(n)}$	۲	$\xrightarrow{f(n)}$	۱۰
۱۰	$\xrightarrow{g(n)}$	۱	$\xrightarrow{f(n)}$	۵

$f(g(x)) = 2 \Rightarrow g(n) = 2 \Rightarrow n = a = 4$

$g(f(x)) = 1 \Rightarrow f(n) = 1 \Rightarrow n = b = 2$

$(a, 2) \Rightarrow a = 4$

$$f \circ f = f(ax+b) = a(ax+b)+b$$

$$f(f(x)) = a^2x + ab + b$$

$$f(x) = -2x - 5 \Rightarrow f(x) = 2x + 1$$

$$\begin{aligned} \xrightarrow{x=-1} f(-1) &= -1 & g(2x+5) &= 2x-5 \Rightarrow 2x+5 = -1 \\ \Rightarrow x &= -3 \Rightarrow g(-1) &= -1 \Rightarrow g \circ f(-1) &= -1 \end{aligned}$$

$$x+|x| \geq 0 \Rightarrow x \geq 0 \quad D_f = [0, +\infty)$$

$$\begin{aligned} x^2 - (x) > 0 &\Rightarrow x(x-1) > 0 \quad x \neq 0, 1 \quad \mathbb{R} - \{0, 1\} \\ x \geq 0 & f(0) = \sqrt{0} = 0 \times \\ f(1) &= \sqrt{1} = 1 \Rightarrow x=1 \\ g \circ f &= \{x \in D_f \mid f(x) \in D_g\} \\ D_{g \circ f} &= (0, 1) \cup (1, +\infty) \end{aligned}$$

$$\begin{aligned} (f+g) \circ f &= \sqrt{x} + \sqrt{1-x^2} = \sqrt{1-x^2} + \sqrt{1-1+x^2} = \sqrt{1-x^2} + |x| \\ \Rightarrow 1-x^2 &\geq 0 \Rightarrow 1 \geq x^2 \Rightarrow -1 \leq x \leq 1 \end{aligned}$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$\begin{aligned} \text{الف)} \quad \frac{2x+1}{x-2} &= t & t(x-2) &= 2x+1 & x(t-2) &= 2t+1 \\ & & & & t &= \frac{2t+1}{t-2} \\ f\left(\frac{2t+1}{t-2}\right) &= \frac{2\left(\frac{2t+1}{t-2}\right)+1}{\frac{2t+1}{t-2}-2} = \frac{4t+2+t-4}{t-2} = \frac{5t-2}{t-2} \end{aligned}$$

$$\begin{aligned} \text{ب)} \quad x + \frac{1}{x} &= t \Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + \frac{2}{x} \Rightarrow \\ \left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) &\longrightarrow t^2 + 2t = f(t) \\ f(x) &= x^2 + \frac{2}{x} \end{aligned}$$

$$g(1) = 0$$

$$g(\sqrt{2}) = 0$$

$$\begin{aligned} f(x) &= x\sqrt{x} \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1 \Rightarrow \sqrt{1} \\ x\sqrt{x} &\Rightarrow x = 2 \Rightarrow \sqrt{2} \end{aligned}$$

$$\text{اختلاف الطرفين} = 2 - 1 = 1$$