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کلیف شماره ۱

بازدهم دخترانه

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$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f(f(\frac{1}{4})) \xrightarrow{\frac{1}{4} < 1} \cot \frac{1}{4} \times \frac{\pi}{4} = \frac{\pi}{4} \Rightarrow \cot \frac{\pi}{4} = \frac{\frac{\pi}{4}}{\frac{1}{4}} = \pi$$

$$\Rightarrow f(\sqrt{\pi}) = \sqrt{\pi^2 + 1} = \pi$$

$$g(\frac{\pi}{4}) = 2 \cos^2 \frac{\pi}{4} = \frac{1}{2} \Rightarrow f(g(\frac{1}{4})) = f(\frac{1}{2}) = \sqrt{\frac{1}{4} - 1} = \frac{\sqrt{3}}{2}$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{\sqrt{2} + 2}{1 - 2} = -\sqrt{2} - 2 \Rightarrow f(-\sqrt{2} - 2) = [-\sqrt{2} - 2]$$

$$\approx [-3, 4] = [-2]$$

$$g(f(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \sqrt{1 - (\frac{\sqrt{2}}{2})^2} = \frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{2} = 1$$

$$الف) f(g(x)) \Rightarrow 1) f \rightarrow 4 \rightarrow 5 \quad 2) 2 \rightarrow f \rightarrow 5 \quad 3) 4 \rightarrow 8 \rightarrow 12 \quad 4) 8 \rightarrow 16 \rightarrow 12$$

$$= \{(4, 5), (2, 5), (4, 12), (8, 12)\}$$

$$ب) g(f(x)) = \emptyset$$

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$$= \{(4, 8), (2, 4), (4, 16)\}$$

$$(f, 2) \in f \circ g \Rightarrow f \rightarrow 3 \rightarrow 2 \Rightarrow a = f$$

$$(f, 1) \in g \circ f \Rightarrow f \rightarrow 5 \rightarrow 1 \Rightarrow b = 5$$

$$\left. \begin{matrix} (a, b) = (f, 5) \end{matrix} \right\}$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = a^2x + ba + b = f(x) \quad -4$$

$$\Rightarrow a^2 = x \Rightarrow a = \pm x, \quad x^2b + b = x \Rightarrow b = 1 \quad \therefore -x^2b + b = x \Rightarrow b = -x^2$$

$$g(x + x^2) = x^2 - x \xrightarrow{x^2 + x^2 = t} x^2 = t - x^2 \Rightarrow x = \frac{t - x^2}{x} \Rightarrow g(x) = \frac{x^2 - x^2}{x} = 0 \quad -5$$

$$\Rightarrow g(x) = \frac{x^2}{x} - \frac{x^2}{x}, \quad f(x) = x^2 + 1 \quad \therefore f(x) = -x^2 - x^2$$

$$f(-1) = -1 \quad \therefore f(-1) = -1 \Rightarrow g(f(-1)) = g(-1) = \frac{-1^2}{-1} - \frac{1^2}{-1} = \frac{-1}{-1} = \frac{-1}{-1} = 1$$

$$D_f \Rightarrow x + |x| \geq 0 \xrightarrow{\text{if } x < 0 \Rightarrow f(x) = 0} \left. \begin{array}{l} \text{if } x \geq 0 \Rightarrow f(x) = 2x \geq 0 \end{array} \right\} D_f = [0, +\infty) \quad -6$$

$$D_g \Rightarrow x^2 - tx \neq 0 \Rightarrow x(x - t) \neq 0 \Rightarrow x \neq 0, x \neq t \xrightarrow{x > 0} \quad -7$$

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \Rightarrow f(x) \in D_g \Rightarrow \sqrt{x + |x|} \neq 0, \sqrt{x + |x|} \neq t$$

$$\stackrel{IC}{\Rightarrow} x + |x| \neq 14 \xrightarrow{\text{if } x < 0 \Rightarrow f(x) = 0 \neq 14 \checkmark}$$

$$\stackrel{IC}{\Rightarrow} D = (0, +\infty) - \{1\}$$

$$\textcircled{v} \text{ if } x > 0 \Rightarrow f(x) = 2x \Rightarrow 2x \neq 14 \Rightarrow x \neq 7$$

$$D_f \Rightarrow 1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f = [-1, 1] \quad -8$$

$$(f + g) \circ f = f(f(x)) + g(f(x)) \Rightarrow \sqrt{1 - x^2} \geq 0 \xrightarrow{\text{for all } x \in \mathbb{R}} \quad -9$$

در دامنه $E = [-1, 1]$ قرار می‌گیرد

$$\stackrel{IC}{\Rightarrow} D = [-1, 1]$$

$$\text{الف) } \frac{x^2 + 1}{x - 2} = t \xrightarrow{x^2 + 1 = tx} (x^2 - t)x = -x^2 - 1 \Rightarrow x = \frac{x^2 + 1}{t - 2} \quad -10$$

$$\Rightarrow f(x) = f\left(\frac{x^2 + 1}{x - 2}\right) + 1 = \frac{1^2x - 11}{x - 2}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \xrightarrow{\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2} x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \quad -11$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \quad \frac{x + \frac{1}{x} = t}{f(t) = t^2 - 2} \Rightarrow f(x) = x^2 - 2x$$

$$g(x\sqrt{x}) \xrightarrow{x\sqrt{x} = x\sqrt{x} \Rightarrow x = 2} g(f(x)) = g(2) \quad -12$$

$$x\sqrt{x} = 1 \Rightarrow x = 1 \xrightarrow{g(f(x)) = g(1)}$$