

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$\rightarrow f \circ f \left(\frac{1}{4} \right) \rightarrow f \left(\frac{1}{4} \right) = \cot \frac{\pi}{16} = \frac{\pi}{4} \rightarrow \cot \frac{\pi}{4} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{3+1} = \sqrt{4} = 2 = f \circ f \left(\frac{1}{4} \right) = 2$$

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الف) $f\left(\frac{1}{n}\right) = \sqrt{\frac{n-1}{n^2}}$, $g(n) = 2 \cos^2 n \rightarrow f(g(\frac{\pi}{4})) \rightarrow g(\frac{\pi}{4}) = 2 \cos^2 \frac{\pi}{4} = 2 \times \frac{1}{2} = 1$

$x=2 \rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{2-1}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

ب) $f(n) = [n]$, $g(n) = \frac{n}{1-n}$, $f \circ g(\sqrt{2}) = ? \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}+1}{1-2} = \frac{\sqrt{2}+1}{-1} = -2-\sqrt{2}$

$f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -2$

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$f(n) = \sin n$, $g(n) = n\sqrt{1-n^2}$

$g \circ f\left(\frac{\pi}{2}\right) \rightarrow f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1 \rightarrow g\left(\frac{1}{2}\right) = \frac{1}{2} \sqrt{1-\frac{1}{4}} = \frac{1}{2} \sqrt{\frac{3}{4}} = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$= \sqrt{\frac{1}{4}} = \frac{1}{2}$

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الف) $f \circ g(n) = f(g(n)) = \{(2, 5), (3, 5), (4, 12), (8, 12)\}$

ب) $g \circ f(n) = g(f(n)) = \{\} = \emptyset$

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د) $g \circ g(n) = g(g(n)) = \{(2, n), (3, 4), (4, 1)\}$

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$f = \{(1, 1), (\underline{3}, \underline{2}), (\underline{4}, \underline{5}), (1, \underline{v})\}$, $g(n) = \{(1, \underline{r}), (\underline{2}, \underline{1}), (\underline{a}, \underline{3}), (\underline{b}, \underline{1})\}$

$(\underline{3}, \underline{2}) \in f \circ g \rightarrow (\underline{3}, \underline{2}) \in f(g(n)) \rightarrow \underline{a} = \underline{3}$

$(\underline{1}, \underline{1}) \in g \circ f \rightarrow (\underline{1}, \underline{1}) \in g(f(n)) \rightarrow \underline{b} = \underline{5}$

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$$f(f(n)) = fn + r \rightarrow a(an+b) + b = fn + r \rightarrow \underbrace{a^2 n}_{a=r} + ab + b = fn + r$$

$$\xrightarrow{a=r} rb + b = r \rightarrow b = 1 \rightarrow f(n) = n + 1$$

$$g(rn + r) = rn - r \rightarrow g \circ f(-1) = ?$$

$$f(-1) = -1 \rightarrow g(-1) \Rightarrow rn + r = -1 \rightarrow g(-1) = -4 - r = -1$$

$$rn = -2$$

$$n = 2$$

$$f(n) = \sqrt{n+1}n, g(n) = \frac{1}{n^2 - 2n}$$

$$D_{g \circ f} = ? \Rightarrow D_{g \circ f} = \left\{ x \in D_f : f(n) \in D_g \right\} \rightarrow \left\{ R : f(n) \neq 0, \varepsilon \right\} \rightarrow D_{g \circ f} = \{ x : x \neq 1 \}$$

$$\rightarrow f(n) \neq 0 \rightarrow x > 0, x \neq 1$$

$$f(n) = \sqrt{1-n^2}, g(n) = \sqrt{n}$$

$$D_{(f+g)(f(n))} \rightarrow D_{f+g} = \left\{ x \in D_f, f(n) \in D_{f+g} \right\} \rightarrow \left\{ 1 \leq x \leq 1, 0 \leq f(n) \leq 1 \right\} \rightarrow D_{(g+f)(f(n))} = [0, 1]$$

$$D_{f+g} = D_f \cap D_g \rightarrow D_{f+g} = \{ x : 0 \leq x \leq 1 \}$$

$$0 \leq \sqrt{1-n^2} \leq 1$$

$$0 \leq 1-n^2 \leq 1$$

$$-1 \leq -n^2 \leq 0$$

$$1 \geq n^2 \geq 0 \rightarrow 1 \geq x \geq 0$$

$$a) f\left(\frac{rn+1}{n-r}\right) = \varepsilon n + \Delta \rightarrow \frac{rn+1}{n-r} = t \rightarrow n = \frac{rt+1}{t-r} \rightarrow f(t) = f\left(\frac{rt+1}{t-r}\right) + \Delta = \frac{1rt-1}{t-r}$$

$$f(t) = \frac{1rt-1}{t-r} \rightarrow f(n) = \frac{1rn-1}{n-r}$$

$$b) f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \rightarrow n + \frac{1}{n} = t \rightarrow f\left(n + \frac{1}{n}\right) = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right) \rightarrow f(t) = \varepsilon - rt$$

$$f(n) = n^r - rn$$

$$g \circ f(n) = g(f(n)) \xrightarrow{f(n)=1, r\sqrt{r}} g(n) = \cdot$$

$$f(n) = \begin{cases} 1 \rightarrow 1 = n\sqrt{n} \rightarrow \boxed{n=1} \\ r\sqrt{r} \rightarrow n\sqrt{n} = r\sqrt{r} \rightarrow \boxed{n=r} \end{cases} \rightarrow |r-1|=1$$