

۱- $f(f(\frac{\sqrt{3}}{2})) \xrightarrow{f(\frac{\sqrt{3}}{2}) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}} f(\frac{\sqrt{3}}{2}) = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

۲- الف) $f(g(\frac{\sqrt{3}}{2})) \rightarrow g(\frac{\sqrt{3}}{2}) = 2 \cos^2 \frac{\pi}{6} = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

ب) $f(g(\sqrt{2})) \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2 \Rightarrow f(-2-\sqrt{2}) = [-2-\sqrt{2}]$
 $= [-3, 4] = -4$

۳- $g(f(\frac{\sqrt{2}}{2})) \rightarrow f(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \Rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

۴- الف) $f(g(x)):$ $\left. \begin{array}{l} 4 \rightarrow 6 \rightarrow 5 \\ 2 \rightarrow 4 \rightarrow 5 \\ 6 \rightarrow 8 \rightarrow 12 \\ 8 \rightarrow 10 \rightarrow 12 \end{array} \right\} \Rightarrow = \{(4,5), (2,5), (6,12), (8,12)\}$

ب) $g(f(x)):$ $\left. \begin{array}{l} 4 \rightarrow 5 \rightarrow 4 \\ 6 \rightarrow 5 \rightarrow 4 \\ 8 \rightarrow 12 \rightarrow 4 \\ 10 \rightarrow 12 \rightarrow 4 \end{array} \right\} \Rightarrow \emptyset$

ج) $f(f(x)):$ $\left. \begin{array}{l} 4 \rightarrow 5 \\ 6 \rightarrow 5 \\ 8 \rightarrow 12 \\ 10 \rightarrow 12 \end{array} \right\} \Rightarrow \emptyset$

د) $g(g(x)):$ $\left. \begin{array}{l} 4 \rightarrow 6 \rightarrow 8 \\ 2 \rightarrow 4 \rightarrow 6 \\ 6 \rightarrow 8 \rightarrow 10 \\ 8 \rightarrow 10 \rightarrow 4 \end{array} \right\} = \{(4,8), (2,6), (6,10)\}$

۵- $f(g(x)):$ $\left. \begin{array}{l} 1 \rightarrow 2 \rightarrow 1 \\ 3 \rightarrow 1 \rightarrow 2 \\ (4,x) \rightarrow a \rightarrow 3 \rightarrow 2 \\ b \rightarrow 1 \rightarrow 2 \end{array} \right\} \Rightarrow a=4 \quad \left. \begin{array}{l} \Rightarrow (a,b) = (4,5) \end{array} \right\}$
 $g(f(x)):$ $\left. \begin{array}{l} 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 2 \rightarrow 1 \\ (4,1) \rightarrow 4 \rightarrow 1 \end{array} \right\} \Rightarrow b=5$

$$f(f(x)) = 5x + 3 \rightarrow ax + b \Rightarrow a(ax + b) + b = a^2x + ba + b = 5x + 3$$

$$g(x) = 3x - 1$$

$$3x + 3 = 4 \Rightarrow x = \frac{4-3}{3}$$

$$g(x) = \frac{3(x-3)}{3} - 1 \rightarrow g(x) = \frac{3x-13}{3}$$

$$f(x) = 3x + 1 \rightarrow -3x - 3$$

$$g(f(-1)) = \frac{f(-1) - 1}{3} \rightarrow g(-1) = \frac{-3 - 1}{3} = \frac{-4}{3} = -1$$

$$x \in \mathbb{R}, x \geq 0 \left\{ \begin{array}{l} x \leq 0 \Rightarrow f(x) = 0 \\ x \geq 0 \Rightarrow f(x) = 3x \end{array} \right\} \Rightarrow D = [0, +\infty)$$

$$x - 5x \neq 0 \Rightarrow x(x-5) \neq 0 \Rightarrow x \neq 0, x \neq 5$$

$$D_{g \circ f(x)} = \{x \in D_f, f(x) \in D_g\} \Rightarrow \sqrt{x+1} \neq 0, \sqrt{x+1} \neq 5$$

$$x+1 \neq 25 \Rightarrow x \neq 24$$

$$f(x) \rightarrow D_f: 1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f = [-1, 1]$$

$$D_g = [0, +\infty)$$

$$D = [-1, 1]$$

$$f(f(x)) + g(f(x)) \rightarrow \sqrt{1-x^2} \geq 0$$

دالة زوجية

$$a) \frac{3x+1}{x-3} = t \rightarrow 3x+1 = t(x-3) \Rightarrow x = \frac{3t+1}{t-3} \quad f(x) = 3\left(\frac{3t+1}{t-3}\right) + 1$$

$$b) \left(\frac{x+1}{x}\right)^3 = x^3 \cdot \frac{1}{x^3} + 3x^2 \cdot \frac{1}{x^2} = \left(x^3 + \frac{1}{x^3}\right) + 3\left(\frac{x+1}{x}\right) \Rightarrow x^3 + \frac{1}{x^3} = \left(\frac{x+1}{x}\right)^3 - 3\left(\frac{x+1}{x}\right)$$

$$\Rightarrow f(x) = x^3 - 3x$$

$$g(f(x)) = g(x\sqrt{x}) \rightarrow x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2 \Rightarrow g(2) = 2-1=1$$

$$x\sqrt{x} = 1 \Rightarrow x = 1 \Rightarrow g(1)$$