$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}\qquad fof\left(\tfrac{2}{3}\right)\Rightarrow \cot\frac{\pi\times 2}{3\times 4}\Rightarrow \cot\frac{\pi}{6}\Rightarrow \sqrt{3}$$

$$\Rightarrow \sqrt{(\sqrt{3})^2+1}\Rightarrow 2$$

$$f\left(\tfrac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}}\qquad g(x)=2\cos^2x\qquad fog\left(\tfrac{\pi}{3}\right)\Rightarrow 2\cos^2\frac{\pi}{3}\Rightarrow \frac{1}{4}\times 2=\frac{1}{2}$$

$$\sqrt{2\left(\tfrac{1}{x}\right)-\tfrac{1}{x^2}}\Rightarrow -\left(\tfrac{1}{2}\right)^2+2\left(\tfrac{1}{2}\right)\Rightarrow \frac{3}{4}\Rightarrow \frac{\sqrt{3}}{2}$$

$$-x^2+2x$$

$$f(x)=[x]\quad g(x)=\frac{x}{1-x}\quad fog(\sqrt{2})\Rightarrow \frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}(1+\sqrt{2})}{1-2}\Rightarrow -\sqrt{2}-2\Rightarrow -4$$

$$f(x)=\sin x\quad g(x)=x\sqrt{1-x^2}\quad gof\left(\tfrac{\pi}{4}\right)\Rightarrow \frac{\sqrt{2}}{2}\Rightarrow 1-\left(\tfrac{\sqrt{2}}{2}\right)^2\Rightarrow \sqrt{\tfrac{1}{2}}\Rightarrow \frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$f(x)=\{(4,5),(6,5),(8,12),(10,12)\}\qquad g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$$

$fog(x)=4\to 6\to 5,\ 2\to 4\Rightarrow 5,\ 6\Rightarrow 8\Rightarrow 12,\ 8\to 10\to 12\Rightarrow\{(4,5),(2,5),(6,12),(8,12)\}$

$gof(x)=4\Rightarrow 5\Rightarrow\times\quad 6\Rightarrow 5\times\quad 8\Rightarrow 12\times\quad 10\Rightarrow 12\times\quad \varnothing$

$fof(x)=4\Rightarrow 5\times\quad 6\Rightarrow 5\times\quad 8\Rightarrow 12\times\quad 10\Rightarrow 12\times\quad \varnothing$

$gog(x)=4\Rightarrow 6\Rightarrow 8,\ 2\Rightarrow 4\Rightarrow 6,\ 6\Rightarrow 8\Rightarrow 10,\ 8\Rightarrow 10\Rightarrow\times\Rightarrow\{(4,8),(2,6),(6,10)\}$

$f=\{(3,1),(2,2),(4,5),(1,7)\}\quad g=\{(1,2),(3,1),(a,3),(b,1)\},\ (4,2)\in fog\quad (a,b)=?$

$(4,1)\in gof\quad (4,5)$

$\underbrace{4\Rightarrow 5\Rightarrow 1}\Rightarrow b=5\qquad 4\Rightarrow ?\Rightarrow 2$

$2\Rightarrow (4,3)\Rightarrow a=4$

g, f خطی $\Rightarrow f(f(x))=4x+3,\ g(2x+3)=3x-2,\ gof(-1)=?\Rightarrow -8$

$a(ax+b)+b\Rightarrow \underbrace{a^2}_{4}x+\underline{ab+b}$

$a=-2\Rightarrow -2b+b=-b=3\Rightarrow -2(-1)-3=-1$

$a=2\Rightarrow 3b=3\Rightarrow 2(-1)+1\Rightarrow -1$

$2x+3=-1$; $2x=-4$; $x=-2\Rightarrow 3(-2)-2=-8$

$f(x)=\sqrt{x+|x|}\ ,\ g(x)=\dfrac{1}{x^2-4x}\quad gof=$ دامنه

$0\ \checkmark,\ -\ \checkmark,\ +\ \checkmark\Rightarrow[0\ +\infty)$

دامنه $\mathbb{R}$

$x(x-4)$; $0\quad 4\Rightarrow \sqrt{x+|x|}\neq 0:\times\Rightarrow \mathbb{R}\cap(0\ +\infty)-\{8\}$

$\sqrt{x+|x|}\neq 4\quad x\neq 8\qquad (0\ +\infty)-\{8\}$

$f(x)=\sqrt{1-x^2}\quad g(x)=\sqrt{x}\quad (f+g)of$ ①

$-1\ 1$ = دامنه = $[-1\ 1]$ ① $\cap[0\ 1]\xrightarrow{\text{دامنه}}\sqrt{1-x^2}+\sqrt{x}\ \checkmark\Rightarrow\cap\Rightarrow[0\ 1]$

$$f\left(\frac{3x+1}{x-2}\right)=4x+5\qquad \frac{3x+1}{x-2}=t\Rightarrow -\frac{-2t-1}{t-3}\Rightarrow f\left(\frac{2t+1}{t-3}\right)+5\Rightarrow\frac{8t+4+5t-15}{t-3}$$

$$\frac{13x-11}{x-3}$$

$$f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}\Rightarrow t^3\Rightarrow x^3+\frac{1}{x^3}+\underbrace{3\frac{x^2}{x}+3\frac{x}{x^2}}\Rightarrow 3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow t^3-3t$$

$$f(x)=x^3-3x$$

اختلاف طول نقاطی که نمودار تابع gof محو x ها را قطع می کند ؟

$g(x) \Rightarrow g(2\sqrt{2}),\ g(1) = 0$

$f(x) = x\sqrt{x} \Rightarrow x\sqrt{x} = 2\sqrt{2} \Rightarrow 2, 1 \Rightarrow 1$

$x\sqrt{x} = 1$