نام و نام خانوادگی محسن جان‌نثاری پاسخنامه تشریحی تکلیف شماره ۹ کلاس یازدهم ریاضی B

۱	$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\leq 1\\ \sqrt{x^2+1} & x>1\end{cases}$ $\quad f\circ f(\frac{2}{3})=f(f(\frac{2}{3}))=f(\cot\frac{\pi}{6})=f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=2$
۲	الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ $\quad g(x)=2\cos^2x$ $\quad f(g(\frac{\pi}{4}))=f(\frac{1}{2})=\sqrt{\frac{4-1}{4}}=\sqrt{\frac{3}{4}}$ $g(\frac{\pi}{4})=\frac{2}{4}=\frac{1}{2}$ ب) $f(x)=\cancel{\sin}[x]$ $\quad g(x)=\frac{x}{1-x}$ $\quad f(g(\sqrt{2}))=$ $g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}(\frac{1+\sqrt{2}}{1+\sqrt{2}})=\frac{\sqrt{2}+2}{-1}=-(\sqrt{2}+2)$ $f(-2-\sqrt{2})=[-\sqrt{2}]-2=-4$
۳	$f(x)=\sin x$ $\quad g(x)=x\sqrt{1-x^2}$ $\quad g(f(\frac{\pi}{4}))=?$ $f(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}}=\frac{\cancel{\sqrt{2}}}{2}\times\frac{1}{\cancel{\sqrt{2}}}=\frac{1}{2}$
۴	$f(x)=\{(4,5)(6,5)(8,12),(10,12)\}$ $\quad g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$ $f\circ g=\{(4,5),(2,5),(6,12),(8,12)\}$ $g\circ f=\emptyset$ $f\circ f=\emptyset$ $g\circ g=\{(2,6)(4,8)(6,10)\}$
۵	$f=\{(2,1)(3,2),(4,5),(1,7)\}$ $\quad g=\{(1,2)(3,1)(a,3)(b,1)\}$ $f(\underbrace{g(4)}_{3})=2$ $\quad g(4)=3$ $\quad \boxed{a=4}$ $g(\underbrace{f(4)}_{5})=1$ $\quad g(5)=1$ $\quad \boxed{b=5}$ $\boxed{(4,5)}$

$f(x)=ax+b$

$f(f(x))=a(ax+b)+b=a^2x+ab+b=4x+3$ $\begin{cases} a^2=4 & a=2 \\ b(a+1)=3 & b=1 \end{cases}$

$f(x)=2x+1$ ۶

$g(2x+3)=3x-2$ $\quad 2x+3=t$, $x=\frac{t-3}{2}$

$g(t)=\frac{3t-9}{2}-2=3t-9-4=3t-13$

$g(f(-1))=g(-1)=\frac{-3-9}{2}-2=-8$

$g(x)=\frac{1}{x^2-4x}$ $\quad f(x)=\sqrt{x+|x|}$ $\quad D_{gof}=\{x\in D_f , f(x)\in D_g\}$

$\begin{array}{ccccc} & 0 & & 4 & \\ + & \vert & - & \vert & + \end{array}$ $\quad D_f=\mathbb{R}$

$D_g=\mathbb{R}-\{0,4\}$ $\quad \underset{(I)}{x\in\mathbb{R}}$, $\sqrt{x+|x|}\neq 0 \rightsquigarrow \mathbb{R}^+$ (II) ۷

$\sqrt{x+|x|}\neq 4 \rightsquigarrow x\neq 8$ (III)

$x+|x|\neq 16$

$(I)\wedge(II)\wedge(III)=\mathbb{R}^+-\{8\}$

$g(x)=\sqrt{x}$ $\quad f(x)=\sqrt{1-x^2}$ $\quad D_{(f+g)of}=\{x\in D_f : f(x)\in D_{f+g}\}$

$D_f=$ $\begin{array}{ccccc} & -1 & & 1 & \\ - & \vert & + & \vert & - \end{array}$ $[-1,1]$ $\quad D_g=[0,+\infty)$

$D_{f+g}=[-1,1]\cap[0,+\infty)=[0,1]$ $\quad D_{f+g}=[0,1]$ ۸

$0\leq\sqrt{1-x^2}\leq 1$

$0\leq 1-x^2\leq 1$ $\quad 0\leq x^2\leq 1$ $\quad 0\leq x\leq 1$

$-1\leq -x^2\leq 0$

الف) $f\left(\frac{3x+1}{x-2}\right)=4x+5$

$\frac{3x+1}{x-2}=t$

$x=\frac{2t+1}{t-3}$

$f(t)=\frac{8t+4}{t-3}+5$

$\boxed{f(x)=\frac{8x+4}{x-3}+5}$

$f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}=$

$\left(x+\frac{1}{x}\right)^3=(t)^3$

$x^3+\frac{1}{x^3}+3(1)\underbrace{\left(x+\frac{1}{x}\right)}_{t}=t^3$ ۹

$x^3+\frac{1}{x^3}=t^3-3t$ $\quad \boxed{f(x)=x^3-3x}$

$g(f(x))=1$,

$g(f(x))=2\sqrt{2}$ $\quad 2-1=1$

$x\sqrt{x}=2\sqrt{2}$ $\quad x=2$ ۱۰

$x\sqrt{x}=1$ $\quad x=1$