

$\frac{1}{\sqrt{2}} < 1 \Rightarrow f\left(\frac{1}{\sqrt{2}}\right) = \cot \frac{\pi \cdot \frac{1}{\sqrt{2}}}{\sqrt{2}} = \cot \frac{\pi}{2} = \cot 90^\circ = \sqrt{2}$ $\rightarrow \sqrt{2} > 1 \Rightarrow f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 - 1} = \sqrt{2-1} = \sqrt{1} = 1 = f \circ f\left(\frac{1}{\sqrt{2}}\right)$	۱
الف) $\frac{1}{x} = t \Rightarrow \frac{1-x}{x^2} = \frac{1}{x} - \frac{1}{x^2} = t - t^2 \Rightarrow f(x) = \sqrt{1-x^2}$ $g\left(\frac{\pi}{\sqrt{2}}\right) = 2 \cos^2 \frac{\pi}{\sqrt{2}} = 2 \left(\frac{1}{\sqrt{2}}\right)^2 = 2 \cdot \frac{1}{2} = 1$, $f\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} = f \circ g\left(\frac{\pi}{\sqrt{2}}\right)$ ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{1-2} = -\sqrt{2}(1+\sqrt{2}) = -\sqrt{2}-2$ $f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4 = f \circ g(\sqrt{2})$	۲
$f\left(\frac{\pi}{\sqrt{2}}\right) = \sin \frac{\pi}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{4}} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} = g \circ f\left(\frac{\pi}{\sqrt{2}}\right)$	۳
الف) $f \circ g(x) = \{(4, 5), (2, 5), (4, 12), (1, 12)\}$ ب) $g \circ f = \emptyset$ $\Rightarrow$ هیچ یک از اعضای برد $f(x)$ در دامنه $g(x)$ نیست ج) $f \circ f = \emptyset$ $\Rightarrow$ هیچ یک از اعضای برد $f(x)$ در دامنه $f(x)$ نیست د) $g \circ g(x) = \{(4, 1), (2, 4), (4, 10)\}$	۴
$f(g(x)) = 2 \Rightarrow g(x) = 2' \Rightarrow x = a$, $x = 4 \Rightarrow a = 4$ $g(f(x)) = 1 \rightarrow f(x) = 4 \Rightarrow$ "۴" برد $f(x)$ و جزو راز $\rightarrow$ غنّ $\rightarrow f(x) = b \Rightarrow x = 4 \rightarrow f(4) = 10 \Rightarrow b = 10$ $(a, b) = (4, 10)$	۵

$$f(x) = ax + b \rightarrow f \circ f(x) = a(ax + b) + b = a^2x + ab + b = kx + r$$

$$\rightarrow a^2 = k \rightarrow a = \pm r$$

$$\textcircled{1} -rb + b = -b = r \Rightarrow b = -r \rightarrow f(x) = -rx + r$$

$$\textcircled{2} rb + b = rb = r \Rightarrow b = 1 \rightarrow f(x) = rx + 1$$

$$rx + r = t \Rightarrow rx = t - r \rightarrow x = \frac{t-r}{r}$$

$$\rightarrow rx - r = r\left(\frac{t-r}{r}\right) - r = \frac{rt-r}{r} - r = \frac{rt-r-r^2}{r} = \frac{rt-r^2-1}{r} = g(t) \Rightarrow g(x) = \frac{rx-r^2-1}{r}$$

$$f(-1) = \frac{(-1)(-r) - r}{r} = \frac{r-r}{r} = 0 \rightarrow g(-1) = \frac{-r-r^2-1}{r} = \frac{-r^2-1-r}{r} = -1 = g \circ f(-1)$$

$$\text{I) } x + |x| \geq 0 \Rightarrow x \in \mathbb{R} \rightarrow D_f = \mathbb{R}, x \in \mathbb{R}$$

$$\text{II) } x^2 - rx = x(x-r) \neq 0 \Rightarrow x \neq 0, r \rightarrow D_g = \mathbb{R} - \{0, r\} \rightarrow f(x) \in D_g$$

$$\text{III) } \sqrt{x+|x|} \in \mathbb{R} - \{0, r\} \Rightarrow \sqrt{x+|x|} \neq 0 \Rightarrow x+|x| \neq 0 \Rightarrow x \leq 0 \rightarrow f(x) = 0$$

$$\sqrt{x+|x|} \neq r \Rightarrow x+|x| \neq r^2 \Rightarrow x \neq r^2$$

$$\Rightarrow x \in (0, r^2) - \{r^2\}$$

$$\text{I} \cap \text{III} = D_{g \circ f} = (0, r^2) - \{r^2\}$$

$$\text{I) } 1-x^2 \geq 0 \Rightarrow \frac{-1}{-2x} \Rightarrow x \in [-1, 1]$$

$$\text{II) } D_{f \circ g} = D_f \cap D_g : D_g = x \geq 0 \rightarrow D_g \cap D_f = [0, 1], f(x) \in [0, 1]$$

$$\text{III) } 0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow 0 \leq 1-x^2 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$$

$$\Rightarrow D_{(f \circ g) \circ f} = [-1, 1]$$

$$\text{الف) } \frac{rx+1}{x-r} = t \rightarrow rx+1 = tx-rt \rightarrow rt+1 = tx-rx = x(t-r) \Rightarrow x = \frac{rt+1}{t-r}$$

$$\rightarrow rx + \omega = r\left(\frac{rt+1}{t-r}\right) + \omega = \frac{rt+r}{t-r} + \omega = \frac{rt+r+\omega t-1\omega}{t-r} = \frac{rt+\omega t-1\omega}{t-r}$$

$$\Rightarrow f(x) = \frac{rt+\omega t-1\omega}{t-r}$$

$$\text{ب) } x + \frac{1}{x} = t \rightarrow t^2 = x^2 + \frac{1}{x^2} + 2\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} + 2t \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2t$$

$$\Rightarrow f(t) = t^2 - 2t \Rightarrow f(x) = x^2 - 2x$$

$$g(f(x)) \rightarrow \text{تفاضل درونی به درونی تابع} \Rightarrow f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow \sqrt{x^3} = 1 \Rightarrow x^3 = 1 \Rightarrow x = 1$$

$$f(x) = x\sqrt{x} \Rightarrow x\sqrt{x} = x\sqrt{x} \Rightarrow \sqrt{x^3} = \sqrt{x} \Rightarrow x^3 = x \Rightarrow x = 1$$

$$\text{اصناف} \\ r-1 = \textcircled{1}$$

نصف به ازای درونی های ۱، ۲، در تابع $g \circ f(x)$ به x ها، مقصود است.