

$$f(f(x)) \rightarrow f(f(\frac{1}{x})) \rightarrow f(\frac{1}{x}) \rightarrow \cos \frac{1}{x} = \cos \frac{\pi}{9} = \sqrt{3}$$

$$f(\sqrt{3}) \rightarrow \sqrt{(\sqrt{3})^2 + 1} \rightarrow \sqrt{4} = 2$$

$$f \circ f(\frac{1}{x}) = 2$$

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$$x = \frac{1}{t} = \frac{t-1}{t} = \frac{t-1}{t} = t - t^{-1} = t + t^{-1} \Rightarrow t + t^{-1} = \pm \sqrt{\frac{t}{t-1}} \\ x = \pm \sqrt{\frac{2}{1-x}} \rightarrow \frac{1}{1+x} \neq \frac{1}{1-x} \rightarrow f \circ g(\frac{x}{1-x}) = \sqrt{\frac{2 \cos^2 x}{1 - 2 \cos^2 x}} \rightarrow \frac{2(\frac{\sqrt{2}}{2})^2}{1 - 2(\frac{\sqrt{2}}{2})^2} = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$$

$$\Rightarrow f \circ g(\sqrt{2}) = \left[\frac{x}{1-x} \right] \rightarrow \left[\frac{\sqrt{2}}{1-\sqrt{2}} \right] \rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} \rightarrow \left[\frac{\sqrt{2}+2}{-1} \right]$$

$$\sqrt{2} = 1.41 \quad [-3.41] = -3$$

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$$\frac{x}{\varepsilon} = 180^\circ \quad \sin \varepsilon = \frac{\sqrt{2}}{2} \\ g(f(x)) = \sin x \sqrt{1 - \sin^2 x} \rightarrow \frac{\sqrt{2}}{2} \sqrt{1 - (\frac{\sqrt{2}}{2})^2} \rightarrow \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} = \frac{1}{2}$$

$$\frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} \rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$\frac{1}{2}$$

$$\text{الف) } f \circ g(x) = \{(x, 1)(1, 2)(2, 3)(3, 4)\}$$

$$\xrightarrow{g \circ f} x \rightarrow y \rightarrow x \rightarrow y \\ \Rightarrow g \circ f(x) = Q$$

$$\text{ج) } f \circ f(x) = Q$$

$$\Rightarrow g \circ g(x) = \{(x, 1)(1, 2)(2, 3)(3, 4)\}$$

$$f \circ g(x) = \{(1, 1)(3, 2)(2, 3)(4, 4)\} \quad g \circ f(x) = \{(1, 2)(2, 3)(3, 4)(4, 1)\}$$

$$1 \rightarrow 2 \rightarrow 1$$

$$a = 1$$

$$2 \rightarrow 1 \rightarrow 2$$

$$3 \rightarrow 2 \rightarrow 3$$

$$4 \rightarrow 3 \rightarrow 4 \Rightarrow 1 \rightarrow 4 \rightarrow 1 \Rightarrow b = 4$$

$$1 \rightarrow 3 \rightarrow 1$$

$$2 \rightarrow 4 \rightarrow 2$$

$$(1, 2) \in f \circ g$$

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$$f(x) = ax + b$$

$$f(f(x)) = a(ax+b) + b = kx + l$$

$$a^2x + ba + b = kx + l$$

$$a^2 = k \quad a = \pm \sqrt{k}$$

$$ba + b = l \xrightarrow{a=\sqrt{k}} b = 1 \Rightarrow \boxed{kx + 1}$$

$$\xrightarrow{a=-\sqrt{k}} b = -1 \Rightarrow \boxed{-kx - 1}$$

$$kx + l = t \Rightarrow x = \frac{t-l}{k}$$

$$k \left(\frac{t-l}{k} \right) - l = \frac{kt - l - kl}{k} = \frac{kt - l(1+k)}{k}$$

$$g(x) = \frac{kx - 1}{k}$$

$$g(f(-1)) = f(-1) \rightarrow k(-1) + 1 = -1$$

$$\rightarrow -k(-1) + l = -1$$

$$g(-1) = \frac{-k - 1}{k} = \frac{-1-k}{k} = \boxed{-1}$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} \rightarrow \{x \in [0, +\infty) \mid \sqrt{x+|x|} \neq \{0, 1\}\}$$

$$f(x) = \sqrt{x+|x|} \quad x+|x| \geq 0$$

$$x \neq 0$$

$$x \neq 1$$

$$|x| \geq -x \rightarrow D_f = [0, +\infty)$$

$$x^k - k \neq 0$$

$$x(x-k) \neq 0$$

$$x \neq 0 \quad x \neq k \rightarrow D_g = \mathbb{R} - \{0, k\}$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2} \quad 1-x^2 \geq 0 \quad x^2 \leq 1 \quad -1 \leq x \leq 1 \quad [-1, 1]$$

$$g(x) = \sqrt{x} \quad [0, +\infty)$$

$$f+g = D_f \cap D_g \rightarrow [0, 1]$$

$$\sqrt{1-x^2} \leq 1 - x^2 \quad \checkmark$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$t = \frac{kx+1}{x-k} \rightarrow tx - kt = kx+1 \quad x(t-k) = kt+1 \quad x = \frac{kt+1}{t-k}$$

$$f(t) = kt+1 \rightarrow k \times \frac{kt+1}{t-k} + 1 \rightarrow \frac{k^2t + k + t - k}{t-k} \rightarrow \frac{k^2t - 1}{t-k} \Rightarrow f(x) = \frac{k^2x - 1}{x-k}$$

$$\Rightarrow x^{\frac{1}{k}} + \frac{1}{x^{\frac{1}{k}}} = \left(x + \frac{1}{x}\right)^{\frac{1}{k}} - \frac{1}{x^{\frac{1}{k}}} \rightarrow t^{\frac{1}{k}} - \frac{1}{t^{\frac{1}{k}}}$$

$$f(t) = t^{\frac{1}{k}} - \frac{1}{t^{\frac{1}{k}}}$$

$$f(x) = x^{\frac{1}{k}} - \frac{1}{x^{\frac{1}{k}}}$$

$$g(1) = 0$$

$$f(x) = x\sqrt{x} = x^{\frac{3}{2}}$$

$$g(\sqrt{x}) = 0$$

$$x^{\frac{1}{k}} = 1 \Rightarrow x = 1$$

$$x^{\frac{1}{k}} = \sqrt[k]{x} \Rightarrow x = y$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$g(y) = \frac{1}{r}$$

$$f\left(\frac{1}{r}\right) = f\left(\frac{1}{a}\right) \Rightarrow a = r$$

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$$f\left(\frac{1}{r}\right) = \frac{\sqrt{r}}{r}$$