

9) الف) $f\left(\frac{3x+1}{x+2}\right) = 4x + 5$

$$\frac{3x+1}{x+2} = t \Rightarrow 3x+1 = t(x+2)$$

$$x(3-t) = -1+2t \Rightarrow x = \frac{-(2-t)}{3-t} = \frac{2-t}{t-3}$$

$$f\left(\frac{2-t}{t-3}\right) + 5 = \frac{12-t}{t-3} + \frac{5t-10}{t-3} = \frac{12-t+5t-10}{t-3} = \frac{4t+2}{t-3} = f(x)$$

ب) $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = x^3 - 3x = f(x)$$

10) $g(1) = 0$, $g(\sqrt{2}) = 0$

$$(g \circ f)(x) = 0 \Rightarrow g(f(x)) = 0$$

$$f(x) = 1 \quad \vee \quad f(x) = \sqrt{2} \quad f(x) = x\sqrt{x} = x \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}} = 1 \Rightarrow x = 1 \quad x^{\frac{3}{2}} = \sqrt{2} = 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{3}{2}} \Rightarrow x = 2$$

$$2-1 = 1$$

$$2) F \circ g \left(\frac{r}{p} \right) = F \left(\frac{1}{r} \right) = \sqrt{\frac{r^2 - 1}{r^2}} \Rightarrow x = r \Rightarrow \sqrt{\frac{r^2 - 1}{r^2}} = \sqrt{\frac{r}{r}} \quad \text{انف}$$

$$\cos \left(\frac{r}{p} \right) = \frac{1}{r} \quad \frac{1}{r} = \frac{1}{r}$$

$$g(x) = r \times \left(\frac{1}{r} \right)^2 = \frac{1}{r}$$

$$F \circ g(\sqrt{r}) \quad \left[\frac{x}{1-x} \right] \Rightarrow \left[\frac{\sqrt{r}}{1-\sqrt{r}} \right] \stackrel{\text{نوع}}{=} [-r - \sqrt{r}] = -r \quad \text{ب.}$$

$$3) F(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g \circ F \left(\frac{r}{p} \right) = \frac{r}{p} \cdot \frac{1}{p}$$

$$g(F(x)) = \sin x \sqrt{1 - \sin^2 x} = \sin x \cos x \Rightarrow x = \frac{r}{p}$$

$$\frac{\sqrt{r}}{p} \times \frac{\sqrt{r}}{p} = \left(\frac{1}{p} \right)$$

$$4) F(F(x)) = rx + p \Rightarrow F(ax+b) \Rightarrow a(ax+b) + b = a^2x + ab + b = rx + p$$

$$a = r \Rightarrow rx + 1$$

$$b = 1$$

$$g(rx+p) = px - r$$

$$g(rx+p) + b = px - r \Rightarrow rax + pa + b = px - r \Rightarrow a = \frac{r}{p} = 1$$

$$y = 1.0x - 4, a \quad b = -4, a$$

$$g \left(\frac{F(-1)}{-1} \right) = -1$$

$$1) F(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad D(F+g) \cap D(F \circ F + g \circ F) \Rightarrow F(F(x)) + g(F(x)) \Rightarrow \sqrt{\sqrt{1-x^2}}$$

$$F(x) = \sqrt{1-x^2} \geq 0 \quad \Downarrow$$

$$D_F = 1-x^2 \geq 0 \quad \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{1-(1-x^2)} = \sqrt{x^2} = |x|$$

$$-1 \leq x \leq 1$$

$$D = [-1, 1]$$