

پاخانامه تشریحی تکلیف شماره 9 کلاس سالیان هجری

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases} \quad f \circ f\left(\frac{1}{4}\right) = ? \quad \frac{1}{4} < 1$$

$$\textcircled{1} \rightarrow \cot \frac{\frac{1}{4} \pi}{4} = \cot \frac{\pi}{16} = \sqrt{2}$$

$$\textcircled{2} \rightarrow \sqrt{2+1} = \sqrt{3} = \textcircled{2}$$

2

$$\frac{\pi}{4} \xrightarrow{f} \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{1}{2} - 1} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$f \circ g\left(\frac{1}{\sqrt{2}}\right) \quad g(x) = 2 \cos^2 x \quad f(x) = \sqrt{\frac{x-1}{2x}}$$

$$\sqrt{2} \xrightarrow{g} \frac{\sqrt{2}}{1-\sqrt{2}}$$

$$\frac{\sqrt{2}}{1-\sqrt{2}} \xrightarrow{f} \left[\frac{\sqrt{2}}{1-\sqrt{2}} \right] \approx [-1, \infty] = [-1, \infty]$$

$$f \circ g\left(\frac{1}{\sqrt{2}}\right) \quad g(x) = \frac{x}{1-x} \quad f(x) = [x]$$

$$\frac{\pi}{4} \xrightarrow{f} \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} \xrightarrow{g} \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\frac{2-\sqrt{2}}{2}} = \frac{1}{\sqrt{2}}$$

$$g \circ f\left(\frac{\pi}{4}\right) = ? \quad g(x) = x\sqrt{1-x^2} \quad f(x) = \sin x$$

$$\Rightarrow g \circ f(x)$$

$$\begin{matrix} f \rightarrow a & x \\ 4 \rightarrow a & x \\ 1 \rightarrow 12 & x \\ 10 \rightarrow 12 & x \end{matrix} \quad \emptyset$$

$$\Rightarrow g \circ g(x)$$

$$\begin{matrix} f \rightarrow 9 \rightarrow 1 \\ 1 \rightarrow 4 \rightarrow 9 \\ 4 \rightarrow 1 \rightarrow 10 \\ 1 \rightarrow 10 \rightarrow x \end{matrix}$$

$$\{(f, 1)(1, 4)(4, 10)\}$$

$$f(x) = \{(f, a)(4, a)(1, 12)(10, 12)\}$$

$$g(x) = \{(f, 9)(1, 4)(4, 1)(10, 10)\}$$

$$\text{الف} \quad f \circ g(x) =$$

$$\{(f, a)(1, a)(4, 12)(10, 12)\}$$

$$\begin{matrix} f \rightarrow 4 \rightarrow a \\ 1 \rightarrow 4 \rightarrow a \\ 4 \rightarrow 1 \rightarrow 12 \\ 1 \rightarrow 10 \rightarrow 12 \end{matrix}$$

$$\text{ب} \quad f \circ f(x)$$

$$\begin{matrix} f \rightarrow a & x \\ 4 \rightarrow a & x \\ 1 \rightarrow 12 & x \\ 10 \rightarrow 12 & x \end{matrix} \quad \emptyset$$

$$f = \{(f, 1)(1, 2)(2, 3)(3, 4)\}$$

$$g = \{(1, 2)(2, 1)(3, 3)(4, 4)\}$$

$$(f, 2) \in f \circ g$$

$$(f, 1) \in g \circ f$$

$$(a, b) = ?$$

$$\text{برای } f \circ g \text{ به } f \text{ می‌رسانیم}$$

$$\text{برای } g \circ f \text{ به } g \text{ می‌رسانیم}$$

$$(f, a)$$

$$f \circ a = f$$

$$g \circ a = a$$

$$b = a$$

2

4

2

5

f و g خطی $f(f(x)) = 7x + 3$ $f(f(x)) = a(ax+b)+b = 7x+3$ $a^2x + ab + b = 7x + 3$ $a^2 = 7 \rightarrow a = \sqrt{7}$ $2b + b = 3 \rightarrow b = 1$ $f(x) = \sqrt{7}x + 1$	$g(7x+3) = 7x-2$ $g(x) = cx+d \rightarrow c(7x+3)+d = 7x-2$ $7cx + 3c + d = 7x - 2$ $7c = 7 \rightarrow c = 1$ $3 \times 1 + d = -2 \rightarrow d = -5$ $g(x) = x - 5$ $g \circ f(-1) = g(f(-1)) = g(\sqrt{7}(-1)+1) = g(-\sqrt{7}+1) = -\sqrt{7}+1-5 = -\sqrt{7}-4$	$g \circ f(-1) = ?$ $g(f(-1)) = g(\sqrt{7}(-1)+1) = g(-\sqrt{7}+1) = -\sqrt{7}+1-5 = -\sqrt{7}-4$
$f(x) = \sqrt{x+1}$ $g(x) = \frac{1}{x^2+1}$ $g \circ f = g(f(x))$ $x \in D_f \rightarrow x+1 \geq 0 \rightarrow [0, +\infty)$ $D_g \rightarrow x^2+1 \neq 0$ $x \neq \pm i$ $D_{g \circ f} = [0, +\infty) \setminus \{1\}$	$f(x) \in D_g$ $x \in (0, +\infty)$ $D_{g \circ f} = (0, +\infty) \setminus \{1\}$	$D_{g \circ f} = ?$
$f(x) = \sqrt{1-x^2}$ $g(x) = \sqrt{x}$ $(f+g) \circ f \rightarrow f \circ f(x) + g \circ f(x) =$ $f \circ f(x) \rightarrow x \in D_f \rightarrow D_f \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$ $f(x) \in D_g \rightarrow f(x) \in D_g \rightarrow -1 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq 1-x^2 \leq 1 \rightarrow -1 \leq x^2 \leq 0 \rightarrow x=0$ $g \circ f(x) \rightarrow x \in D_f \rightarrow -1 \leq x \leq 1$ $f(x) \in D_g \rightarrow \sqrt{1-x^2} \geq 0 \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$ $D_{(f+g) \circ f} \rightarrow [-1, 1]$	$D_{(f+g) \circ f} = ?$	$D_{(f+g) \circ f} = ?$
$f(x) = \frac{x+1}{x-2}$ $f(t) = \frac{t+1}{t-2} + \omega = \frac{-1t-1}{t-2} + \omega = \frac{-1t-1+\omega(t-2)}{t-2} = \frac{-1t-1+\omega t-2\omega}{t-2} = \frac{(\omega-1)t-1-2\omega}{t-2}$ $f(x) = \frac{x+1}{x-2}$ $x + \frac{1}{x} = t \rightarrow (x + \frac{1}{x})^2 = x^2 + 2 + \frac{1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$ $f(t) = t^2 - 2$ $f(x) = x^2 - 2$	$f(x) = \frac{x+1}{x-2}$ $x + \frac{1}{x} = t \rightarrow (x + \frac{1}{x})^2 = x^2 + 2 + \frac{1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$ $f(t) = t^2 - 2$ $f(x) = x^2 - 2$	$f(x) = ?$ $f(x) = \frac{x+1}{x-2}$
$g \circ f(x) = g(f(x))$ $if g(f(x))$, $f(x) = 1 \rightarrow g(f(x)) = 0 \rightarrow if f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$ $if g(f(x))$, $f(x) = \sqrt{2} \Rightarrow g(f(x)) = 0 \rightarrow if f(x) = \sqrt{2} \Rightarrow x\sqrt{x} = \sqrt{2} \Rightarrow x = 2$ $2-1 = 1$	$g \circ f(x) = g(f(x))$ $if g(f(x))$, $f(x) = 1 \rightarrow g(f(x)) = 0 \rightarrow if f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$ $if g(f(x))$, $f(x) = \sqrt{2} \Rightarrow g(f(x)) = 0 \rightarrow if f(x) = \sqrt{2} \Rightarrow x\sqrt{x} = \sqrt{2} \Rightarrow x = 2$ $2-1 = 1$	$g \circ f(x) = ?$