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$h(x) = 2g(x) - 3f(x)$ $g(x) = \{(-1, 1), (0, 4), (1, 0), (2, 1)\}$ و $f(x) = \sqrt{1-x^2}$

$2g = \{(-1, 2), (0, 8), (2, 0), (4, 2)\}$ $\rightarrow R_h = \{2, 0, 1, 4\}$

$3f = \{(-1, 0), (0, 3), (1, \sqrt{3}), (2, 0)\}$

$R_h = \{2+0+1+0=11\} \Rightarrow \text{مجموع درجتها} = 11$

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$f(x) = 2x-1$ $D_f = [3, +\infty)$ $g(x) = \frac{1}{x} x+3$ $D_g = (-\infty, 3]$

$R_f \cup R_g \Rightarrow R_f = [5, +\infty)$ و $R_g = (-\infty, 5]$

از جایگذاری استفاده می‌کنیم $\Rightarrow (-\infty, 5] \cup [5, +\infty)$

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$y = \frac{-x^2}{2} + x + 3$ $\sqrt{b-a} \Rightarrow \text{بیشترین مقدار}$

$-\frac{x^2}{2} + x + 3 > \frac{3}{2} \Rightarrow -x^2 + 2x + 9 > 3 \Rightarrow x^2 - 2x - 3 < 0$

$(x-4)(x+1) < 0$

$\begin{matrix} -1 & 4 \\ a & b \end{matrix}$ $\begin{matrix} + & - & + \end{matrix}$

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$y = |x-1| + |x-2| + |x-3|$

کمترین مقدار $|x-2| \Leftarrow$

$\begin{cases} 1-x+1-x+3-x-4 & x < 1 \\ x-1+3-x+3-x-4 & 1 \leq x < 2 \\ x-1+3-x+2x-4 & 2 \leq x < 3 \\ x-1+x-3+2x-4 & x \geq 3 \end{cases}$

مقدار $\rightarrow x=1 \Rightarrow y=5$
 $\rightarrow x=2 \Rightarrow y=2$
 $\rightarrow x=3 \Rightarrow y=5$

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$y = |x| - 2|x+1|$

$\begin{cases} x+2 & x < -1 \\ -3x-2-1 & -1 \leq x < 0 \\ -x-2 & x \geq 0 \end{cases}$ $R_y = (-\infty, 1]$

$$y = \frac{x^2 + \omega x + m}{x+1}$$

* برای هر x هیچ مقدار طبیعی و غیر طبیعی برداشتی توان R باشد و همیشه $R - \{-1\}$ خواهد بود

$$x \neq -1 \Rightarrow \left(\frac{1}{x+1}\right)^{-\omega} + (x+1)\omega + m = 0$$

$$m = 6$$

$$\text{فرض} \Rightarrow x^2 + \omega x + 6 \Rightarrow \frac{(x+1)(x+6)}{x+1} \Rightarrow y = x+6 \Rightarrow (بر: x \neq -1)$$

$$R_y = R - \{-1\}$$

$$f(x) = \begin{cases} x+2 & x \geq 3 \\ x^2 - 2x + 2 & 0 \leq x < 3 \\ 1-x+2 & x < 0 \end{cases}$$

$$R_f = [\omega + \infty) \cup [1, \omega) \cup (2, \omega + \infty) \Rightarrow [1, \omega + \infty)$$

$$\left. \begin{array}{l} 1) x \geq 3 \Rightarrow x+2 \Rightarrow y = [\omega + \infty) \\ 2) (x-1)^2 + 1 \Rightarrow 0 \leq x < 3 \Rightarrow y \in [1, \omega) \\ 3) |x|+2 \Rightarrow x < 0 \Rightarrow y \in (2, \omega + \infty) \end{array} \right\} \text{جایگذاری نقاط}$$

$$f(x) = \begin{cases} x^2 + 6x + 2 & x \leq 0 \\ [2x] - 2x & x > 0 \end{cases} \quad [2x] - 2x \Rightarrow 2([2x] - x) \\ R_y = [-1, 0)$$

$$x = \frac{-b}{2a} = -2 \Rightarrow \min \quad \cup \quad R_f = [-1, \omega + \infty)$$

$$y = (-2)^2 + (-2) \leq + 2 = -1 \Rightarrow R_y = [-1, \omega + \infty)$$

$$y = a+1 - \sqrt{2x+3} \quad D_y = [b, \omega + \infty) \quad R_y = (\omega + \infty) \quad a, b \in \mathbb{Z}$$

$$2x+3 \geq 0 \Rightarrow 2x \geq -3 \Rightarrow x \geq -\frac{3}{2} \Rightarrow y_{\min} \Rightarrow a+1 - \sqrt{2(-\frac{3}{2})+3} = a+1$$

$$a+1 = \omega \Rightarrow a = 6$$

$$(b = -\frac{3}{2}) \quad a, b \in \mathbb{Z} \Rightarrow x - \frac{3}{2} = -6$$

$$f(x) = 2\sqrt{x+1} + 5\sqrt{1-x} \quad f(x) + g(x) = 3\sqrt{2+2\sqrt{1-x^2}} \quad D_f = D_g$$

$$R_y \Rightarrow \frac{f(x)}{9} - \frac{g(x)}{3}$$

همه اعداد حقیقی مساوی

$$g = (f+g) - f \Rightarrow y = \frac{f}{9} - \frac{g}{3} = \frac{f}{9} - \frac{(f+g)-f}{3} = \frac{f}{9} - \frac{f+g}{3} + \frac{f}{3}$$

$$\Rightarrow \frac{f}{9} - \frac{f+g}{3} \Rightarrow y = \frac{1}{9}(2\sqrt{x+1} + 5\sqrt{1-x}) - \frac{1}{3}(3\sqrt{2+2\sqrt{1-x^2}}) \Rightarrow \sqrt{x+1} + 5\sqrt{1-x} - \sqrt{2+2\sqrt{1-x^2}}$$

$$x \in [-1, 1] \Rightarrow R_y = [0, \sqrt{2}]$$

$$x^2 + (a-y)x + m - y = 0 \Rightarrow x = \frac{y - a \pm \sqrt{(a-y)^2 - 4(m-y)}}{2}$$

$$ay = 0 \rightarrow 1 > 0$$

$$\Delta < 0 \rightarrow m < 4 \rightarrow$$

$$m = 1, 2, 3$$

مضرب نه باشد

$$y^2 - 4y + 12 - 4m > 0$$