سایا جلوری

①

$1-x^2 \geq 0 \longrightarrow x^2 \leq 1 \longrightarrow -1 \leq x \leq 1 \longrightarrow D_f = [-1,1]$ ①

$R_f = \{0,1\}$

$R_g = \{1,2,4\}$

$D_g = \{-1,0,1,2\}$ ②

① ∩ ② = $\{-1,0,1\}$

$y_4 - 3f = \begin{cases} 2 = 0 \times 2 - 3^2 \times 2 \\ 0 = 1 \times 2 - 3^2 \times 4 \\ -2 = 0 \times 2 - 3^2 \times 1 \end{cases} \Rightarrow$ ④

②

$f(x) = 2x - 1 \longrightarrow D_f = [3, +\infty) \longrightarrow f(3) = 6 - 1 = 5$

$g(x) = \frac{1}{2}x + 3 \longrightarrow D_g = (-\infty, 3] \longrightarrow g(3) = \frac{3}{2} + 3 = \frac{9}{2}$

$R_f = [0, +\infty)$

$R_g = (-\infty, \frac{9}{2}]$ $\cup \Rightarrow R - (\frac{9}{2}, 0)$

③

$y = \frac{-x^2}{2} + x + 3 \longrightarrow \left(\frac{y}{2} = -\frac{x^2}{2} + x + 3\right) \times 2 \longrightarrow y = -x^2 + 2x + 6 \longrightarrow x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$, $x = -1$ $\longrightarrow (-1, 3)$

$\sqrt{b-a} \Rightarrow \sqrt{3-(-1)} = \sqrt{4} = 2$

④

$y = |x-1| + |x-3| + |2x-4|$

$x = 1 \longrightarrow y = 4$

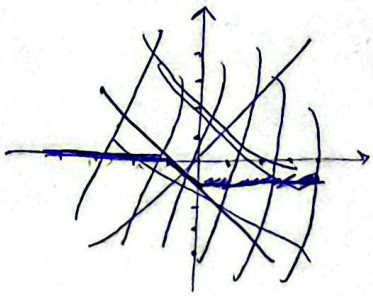
$x = 2 \longrightarrow y = 2$ ← کمترین مقدار

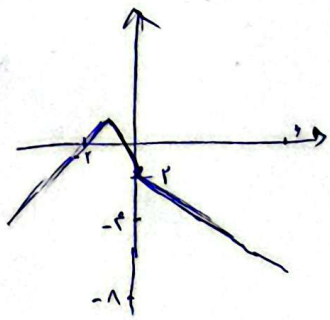
$x = 3 \longrightarrow y = 4$

⑤

$y = |x| - 2|x+1|$

$$\begin{cases} x+2 & x < -1 \\ -3x - 2 & -1 \leq x < 0 \\ -x - 2 & x \geq 0 \end{cases}$$





⑥

$y = \frac{x^2 + 5x + m}{x+1} = 2 \Rightarrow x^2 + (5-2)x + (m-2) = 0$

$\Delta = y^2 - 4y + (20 - 4m) > 0 \longrightarrow \Delta = (-4)^2 - 4(1)(20-4m) = 36 - 80 + 16m = 16m - 44$

$\Delta \leq 0 \longrightarrow 16m - 44 \leq 0 \longrightarrow m \leq 4$

$m = 4 \longrightarrow y = \frac{x^2 + 5x + 4}{x+1} = \frac{(x+1)(x+4)}{x+1} \longrightarrow x \neq -1 \Rightarrow y = x + 4$

$x = -1 \Rightarrow y = -1 + 4 = 3 \longrightarrow m = 4 \xrightarrow{R} R - \{3\} \longrightarrow m < 4 \longrightarrow \{1,2,3\} = D_{new}$

(۷)

$$f(x)=\begin{cases} x+1 & x\geq 2 \\ x^2-2x+2 & 0\leq x<2 \\ |x|+2 & x<0 \end{cases}$$

$x<0 \Rightarrow f(x)=|x|+2=-x+2$ ، $|x|>0$ یا $x<0$

$f(x)>0$

$R=(2,+\infty)\cup$

۲) $0\leq x<2 \rightarrow f(x)=x^2-2x+2=(x-1)^2$

$x=1 \rightarrow y=1$

$x=0 \rightarrow y=2$ $\Rightarrow$ $[1,2)\cup$ $\Rightarrow$ $[1,+\infty)$

$x=2 \rightarrow$ $4-4+2$

۳) $f(x)=x+2 \quad x\geq 2$

$x=2 \rightarrow f(x)=4 \rightarrow R=[4,+\infty)\cup$

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(۸)

$$f(x)=\begin{cases} x^2+4x+2 & x\leq 0 \\ [2x]-2x & x>0 \end{cases}$$

$x\leq 0 \rightarrow f(x)=x^2+4x+2=(x+2)^2-2$

$f(-2)=x^2+4x+2=4-8+2=-2 \rightarrow \min$

$x=0 \rightarrow f(0)=2 \rightarrow R=[-2,+\infty)$

$F(x)=[2x]-2x$

$x>0 \rightarrow n=[2x] \rightarrow n\leq 2x<n+1$

$f(x)=n-2x$ $(-1,0)$ $\rightarrow$ $(-1,0)$

$[-2,+\infty)\cup(-1,0)=[-2,+\infty)$

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(۹)

$y=a+1-\sqrt{2x+2}$

$b=x=-\frac{2}{2}$ $\Rightarrow$ $0=a+1 \rightarrow a=-1$ $\rightarrow$ $-\frac{2}{2}\times -1=-y$

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(۱۰)

$f(x)=2\sqrt{x+1}+2\sqrt{1-x} \rightarrow x=-1$, $x=1$

x	-1		1
	$-$	$+$	$-$

$D_f=[-1,1]$

$f(x)g(x)=2\sqrt{2+2\sqrt{1-x^2}}$

$g(x)=2\sqrt{2+2\sqrt{1-x^2}}\cdot f(x)^{-1}$ $\forall x\in[-1,1]$ $\rightarrow$ $D_f=D_g$

$y=\frac{f(x)}{4}-\frac{g(x)}{2} \Rightarrow \frac{f(x)}{4}-\frac{1}{2}\times\left(2\sqrt{2+2\sqrt{1-x^2}}-f(x)\right)\Rightarrow$

$\frac{4f(x)}{4}-\sqrt{2+2\sqrt{1-x^2}}+\frac{2\times f(x)}{2\times 2} \rightarrow \frac{2\times f(x)}{2\times 2}-\sqrt{2+2\sqrt{1-x^2}} \Rightarrow \sqrt{1+x}+2\sqrt{1-x}\sqrt{2-2\sqrt{1-x^2}}$

$1\Rightarrow \sqrt{2}-\sqrt{2}=0$ $\quad -1 \rightarrow 2\sqrt{2}-\sqrt{2}=\sqrt{2}$

$R=[0,\sqrt{2}]$