

ا) $r \sin x + r \cos x = y \Rightarrow -\sqrt{a^2+b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2+b^2} \Rightarrow B_{\text{مركزه}} - \text{نقطه}$
 $\hookrightarrow -\sqrt{9+4} \leq r \sin x + r \cos x \leq \sqrt{9+4} \Rightarrow R_y = [-\sqrt{13}, \sqrt{13}]$

ب) $y = r \sin x - r \cos x \Rightarrow -\sqrt{r^2+r^2} \leq r \sin x - r \cos x \leq \sqrt{r^2+r^2} \Rightarrow R_y = [-\sqrt{2}, \sqrt{2}]$

ج) $y = [r \sin x + \omega \cos x] \Rightarrow -\sqrt{r^2+\omega^2} \leq r \sin x + \omega \cos x \leq \sqrt{r^2+\omega^2} \Rightarrow [-\sqrt{r^2+\omega^2}, \sqrt{r^2+\omega^2}] \Rightarrow R_y = [-\sqrt{r^2+\omega^2}, \sqrt{r^2+\omega^2}]$
 $\hookrightarrow = \{x \mid x \in \mathbb{Z}, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\}$

د) $y = \sqrt{\cos x - r \sin x} \Rightarrow -\sqrt{r^2+1} \leq \cos x - r \sin x \leq \sqrt{r^2+1}$
 $\Rightarrow \sqrt{[-\sqrt{r^2+1}, \sqrt{r^2+1}]} \Rightarrow R_y = [0, \sqrt{r^2+1}]$

هـ) $y = \sin^2 x + r \sin x + r \Rightarrow R_y = [-\frac{1}{r}, 4]$
 $-\frac{b}{2a} = -\frac{r}{2} \Rightarrow \min \Rightarrow f(-\frac{r}{2}) = -\frac{1}{r}$
 $\begin{cases} \sin x = -1 \rightarrow 1 - r + r = 0 \\ \sin x = 1 \rightarrow 1 + r + r = 4 \rightarrow \max \end{cases}$
 ب) $r \sin^2 x + r \sin x + r \Rightarrow R_y = [\frac{r}{\lambda}, r]$
 $-\frac{b}{2a} = -\frac{r}{2} \rightarrow f(-\frac{r}{2}) = \frac{r}{\lambda}$
 $\begin{cases} \sin x = -1 \rightarrow r - r + r = r \rightarrow \min \\ \sin x = 1 \rightarrow r + r + r = 3r \rightarrow \max \end{cases}$

و) $y = \sin^2 x + r \cos^2 x = r \cos^2 x + (1 - \cos^2 x) = r \cos^2 x + 1 \Rightarrow R_y = [1, r]$
 $[0, 1] \rightarrow [0, r] \xrightarrow{+1}$

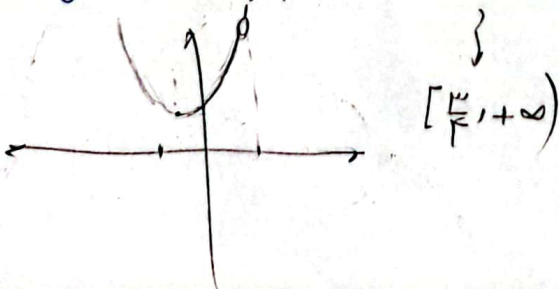
$y = \frac{r \cot^2 x}{1 + \cot^2 x} = \frac{r \cos^2 x}{\sin^2 x} \times \sin^2 x = r \sin x \cos x = \sin 2x \Rightarrow R_y = [-1, 1]$

ز) $\sin^2 x + \cos^2 x = y \Rightarrow r^{1-k} \leq \sin^2 x + \cos^2 x \leq 1 \Rightarrow r^{1-k} \leq \sin^2 x + \cos^2 x \leq 1$
 $\Rightarrow \frac{1}{r} \leq \sin^2 x + \cos^2 x \leq 1 \Rightarrow R_y = [\frac{1}{r}, 1]$

ح) $y = [\sin^2 x + \cos^2 x] \Rightarrow \left[\frac{1}{r} \leq \sin^2 x + \cos^2 x \leq 1 \right] \Rightarrow R_y = [\frac{1}{r}, 1]$

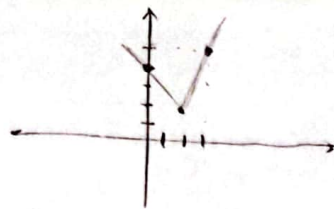
ط) $y = \frac{r x^2 + \lambda x - r}{x + r} \rightarrow \frac{(r x - 1)(x + r)}{x + r} = r x - 1 \Rightarrow R_y = [R - 1, R]$

ث) $\frac{x^2-1}{x-1} = y \rightarrow \frac{(x+1)(x-1)}{x-1} = x+1 \Rightarrow R_y = [R, +\infty)$



$$a) y = x + |x-4|$$

$$\begin{array}{c|c|c} -x+4 & x & x-4 \\ \hline | & 0 & | \\ 4 & 4 & 4 \end{array}$$



باز هم (ضرب)

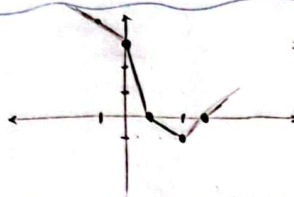
انبار (تربیع) - تطبیق

$$\Rightarrow R_y = [4, +\infty)$$

مسئله

$$b) y = |x-2| + |x-1| - |x|$$

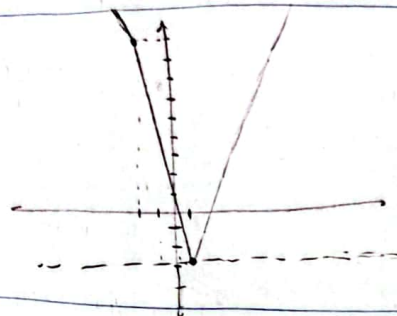
$$\begin{array}{c|c|c|c|c} -x+2 & -x+1 & -x & x-2 & x-1 \\ \hline | & 0 & | & 0 & | \\ 2 & 1 & 0 & 2 & 1 \end{array}$$



$$\Rightarrow R_y = [-1, +\infty)$$

$$|x-2| - |x+1| = k$$

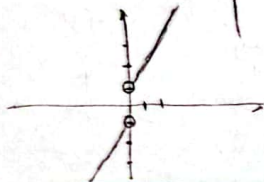
$$\begin{array}{c|c|c} -x+2 & x & x+1 \\ \hline | & 0 & | \\ 2 & 2 & 2 \end{array}$$



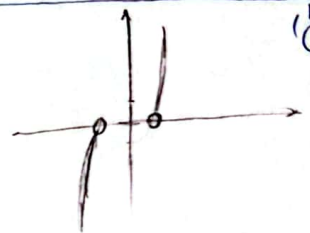
$$\Rightarrow x=1 \rightarrow |x-2| - |x+1| = \boxed{-3} = k$$

مسئله

$$c) y = x + \frac{x}{|x|} \begin{cases} x+1 : x > 0 \\ x-1 : x < 0 \end{cases}$$



$$b) x|x| - \frac{x}{|x|} = \begin{cases} x^2-1 & x > 0 \\ -x^2+1 & x < 0 \end{cases}$$



مسئله

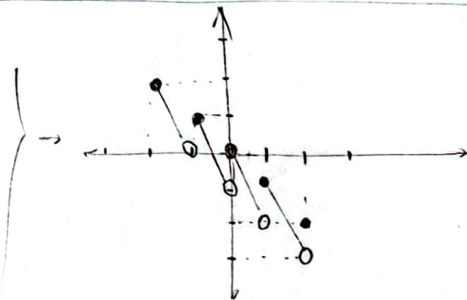
$$d) y = [x] - 2x \Rightarrow \Delta y = \Delta x \rightarrow y = -x$$

$$\Delta y = [1, 2) \rightarrow y = -2x+1$$

$$\Delta y = [0, 1) \rightarrow y = -2x$$

$$\Delta y = [-1, 0) \rightarrow y = -2x-1$$

$$\Delta y = [-2, -1) \rightarrow y = -2x-2$$



مسئله

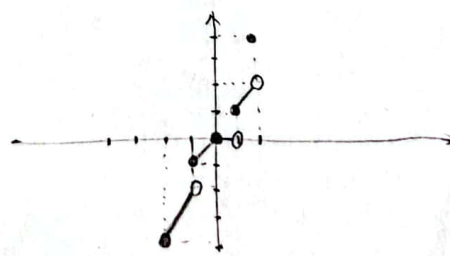
$$b) y = [x] |x| \Rightarrow \Delta y = \Delta x \rightarrow y = x$$

$$\Delta y = [1, 2) \rightarrow y = x$$

$$\Delta y = [0, 1) \rightarrow y = 0$$

$$\Delta y = [-1, 0) \rightarrow -1|x| \rightarrow x$$

$$\Delta y = [-2, -1) \rightarrow -2|x| \rightarrow x$$



$$a) y = x - [x] \rightarrow 0 \leq x - [x] < 1 \rightarrow R_y = [0, 1)$$

مسئله

$$b) y = x - x[x] \rightarrow 0 \leq x - [x] < 1 \Rightarrow 0 \leq x - [x] < 1 \Rightarrow R_y = [0, 1)$$

$$c) y = x - \omega\left(\frac{x}{\omega}\right) \rightarrow \omega\left(\frac{x}{\omega} - \left[\frac{x}{\omega}\right]\right) \Rightarrow 0 \leq \omega\left(\frac{x}{\omega} - \left[\frac{x}{\omega}\right]\right) < \omega \rightarrow R_y = [0, \omega)$$

$$d) y = [x] - 2x \rightarrow 0 \leq [x] - 2x < 1 \Rightarrow 0 \leq [x] - 2x < 1 \rightarrow R_y = [-1, 0]$$