توابع و خواص آنها

۸۰

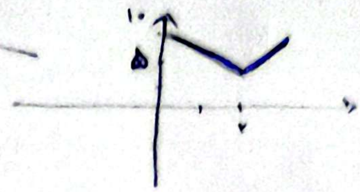
متعلم یازدهم B ساسی سرشته

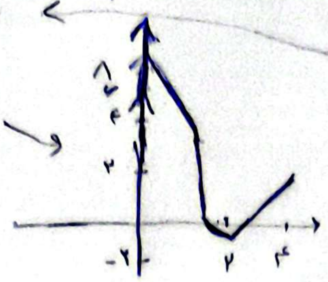
نام و نام خانوادگی: ساینا حاوری

۱)

الف) $y = x + |2x-4|$

ب) $y = |x-2| + |x-1| - x$





۲)

$|3x-3| - |x+2| = K$

$x = 2 \rightarrow K = 9$

$x = 1 \rightarrow 3(1)-3 - |3| = K \rightarrow K = -3 \rightarrow$ کمترین مقدار

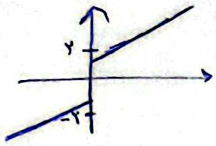
$K > -3 \rightarrow y = K \rightarrow$ ۲ جواب، دو بار و قطع می‌کند

$K = -3 \rightarrow$ یک جواب $\rightarrow x = 1 \rightarrow \boxed{K = -3}$

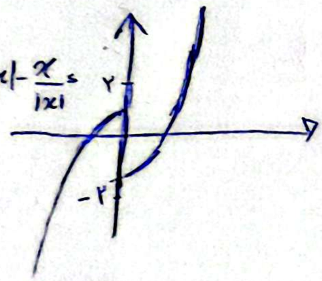
$K < -3 \rightarrow$ هیچ جوابی ندارد

۳)

الف) $x + \frac{x}{|x|} =$

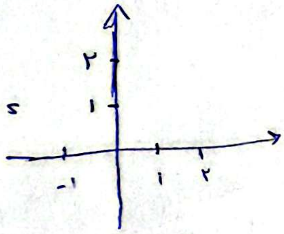


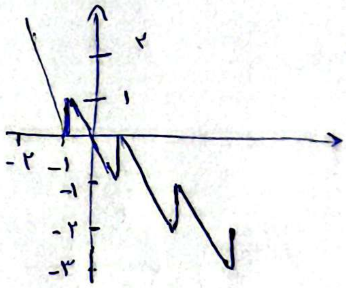
ب) $x|x| - \frac{x}{|x|} =$



۴)

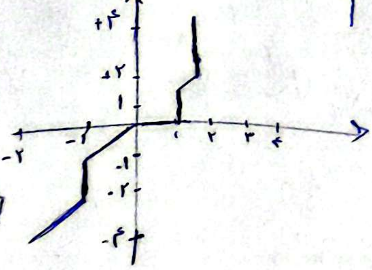
الف) $y = [x] - 2x =$





در بازه $[-2, 2]$

ب) $y = [x][x] =$



۵)

① $y = 3x - [3x]$

$\rightarrow 0 \le y < 1 \rightarrow R_f = [0, 1)$

② $y = 4x - 4[x]$

$\rightarrow 0 \le y < 4 \rightarrow R_f = [0, 4)$

③ $y = x - 5\left[\frac{x}{5}\right]$

$\rightarrow 0 \le y < 5 \rightarrow R_f = [0, 5)$

④ $[2x] - 2x = -1 < y \le 0 \rightarrow R_f = (-1, 0]$

$y = 3\sin x + 2\cos x$

$a = 3 \quad b = 2$

$[-\sqrt{3^2+2^2}, \sqrt{3^2+2^2}] , R_f = [-\sqrt{13}, \sqrt{13}]$

(6) $y = a\sin(x) + b\cos(x) \to R_f = [-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$

$y = 4\sin x - 3\cos x \to [-\sqrt{16+9}, \sqrt{16+9}]$

$R_f = -5 \leq y \leq 5 \to [-5, 5]$

$y = [2\sin x + 5\cos x]$

$R = \sqrt{2^2+5^2} = \sqrt{29} \to -\sqrt{29} \leq 2\sin x + 5\cos x \leq \sqrt{29}$

$[-5.4, 5.4]$

اعدادی در بازه هستند $= \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

$y = \sqrt{\cos x - 2\sin x} \to \cos x - 2\sin x \geq 0$

$R = \sqrt{1^2+(-2)^2} = \sqrt{5} \to -\sqrt{5} \leq \cos x - 2\sin x \leq \sqrt{5}$

$0 \leq y \leq \sqrt{\sqrt{5}} \to R_f = [0, \sqrt{\sqrt{5}}]$

(7)

الف) $y = \sin^2 x + 3\sin x + 2 \to y = t^2 + 3t + 2 = (t+1)(t+2) \to t = -\frac{3}{2}$

$y(-1) = (-1)^2 + 3(-1) + 2 = 0 , \quad y(1) = 1 + 3 + 2 = 6 \to [0, 6]$

ب) $y = 2\sin^2 x + 3\sin x + 2$

$y = 2t^2 + 3t + 2 \to t = -\frac{3}{4}$

$y(-\frac{3}{4}) = 2(\frac{9}{16}) + 3(-\frac{3}{4}) + 2 = \frac{18}{16} - \frac{9}{4} + 2 = \frac{9}{8} - \frac{18}{8} + \frac{16}{8} = \frac{7}{8}$

$y(-1) = 2(1) + 3(-1) + 2 = 1 \quad y(1) = 2(1) + 3(1) + 2 = 7 \to R_f = [\frac{7}{8}, 7]$

(8)

$y = \sin^2 x + 4\cos^2 x$

$\sin^2 x + \cos^2 x = 1 \to \cos^2 x = 1 - \sin^2 x \to y = \sin^2 x + 4 - 4\sin^2 x$

$4 - 3\sin^2 x$

$0 \leq \sin^2 x \leq 1 \to 4 + 0 \geq 4 - 3\sin^2 x \geq 4 - 3 \to 4 \geq y \geq 1 \to [1, 4]$

$y = \frac{2\cot x}{1+\cot^2 x} \to 1 + \cot^2 x = \csc^2 x \to \csc x = \frac{1}{\sin x} \quad \cot x = \frac{\cos x}{\sin x}$

$y = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} \Rightarrow y = \frac{2\cos x}{\sin x} \times \sin^2 x = 2\cos x \times \sin x$

$2\sin x \cos x = \sin(2x) \to [-1, 1]$

(9)

الف) $y = \sin^4 x + \cos^4 x = (\sin^2 x)^2 + (\cos^2 x)^2 \qquad a = 0 \to y = 1$

$a = \sin^2 x \to 0 \leq a \leq 1 \to \cos^2 x = 1 - \sin^2 x \quad (a = \sin^2 x) \quad a = 1 \Rightarrow y = 1$

$y = 2a^2 - 2a + 1 , \quad 0 \leq a \leq 1 \to a = \frac{-(-2)}{4} = \frac{2}{4} = \frac{1}{2} \qquad R_f = [1, \frac{1}{2}]$

$y = 2(\frac{1}{2})^2 - 2(\frac{1}{2}) + 1 = \frac{2}{4} - \frac{2}{2} + 1 = \frac{1}{2}$

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ب) $[\sin^8 x + \cos^8 x]$ (۹)

$\min = \frac{1}{8}$
$\max = 1 \longrightarrow \{\frac{1}{8}, 1\} \longrightarrow R_f = [0, 1]$

الف) $y = \dfrac{2x^2 + 7x - 4}{x + 4} \rightarrow x \neq -4 \rightarrow \div \; x + 4$ (۱۰)

$= 2x - 1 \longrightarrow x \neq -4 \longrightarrow y = 2(-4) - 1 = -9$

$R_f = \{-9\}$ $\quad x = -4$

ج) $y = \dfrac{x^3 - 1}{x - 1} \Rightarrow \dfrac{(x-1)(x^2 + x + 1)}{x - 1}$

$y = x^2 + x + 1 \quad x \neq 1$

$x = -\dfrac{b}{2a} = -\dfrac{1}{2} \longrightarrow y(-\frac{1}{2}) = (-\frac{1}{2})^2 + (-\frac{1}{2}) + 1 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$

$R_f = [\frac{3}{4}, +\infty)$

$x = 1 \quad y = 1^2 + 1 + 1 = 3$

$[\frac{3}{4}, +\infty) - \{3\}$