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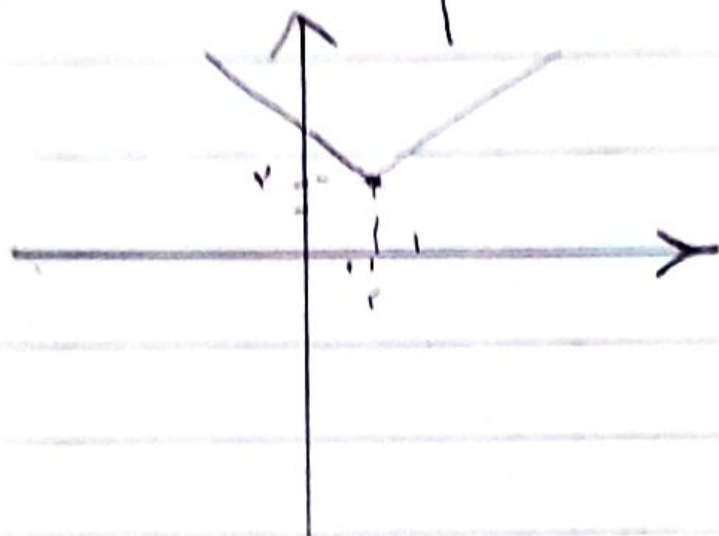
19

معادله: $x + |x-2| = 1$

مجموعه جوابات: $\{x \mid x \geq 2\}$

معادله: $x + |x-2| = 1$

مجموعه جوابات: $\{x \mid x < 2\}$



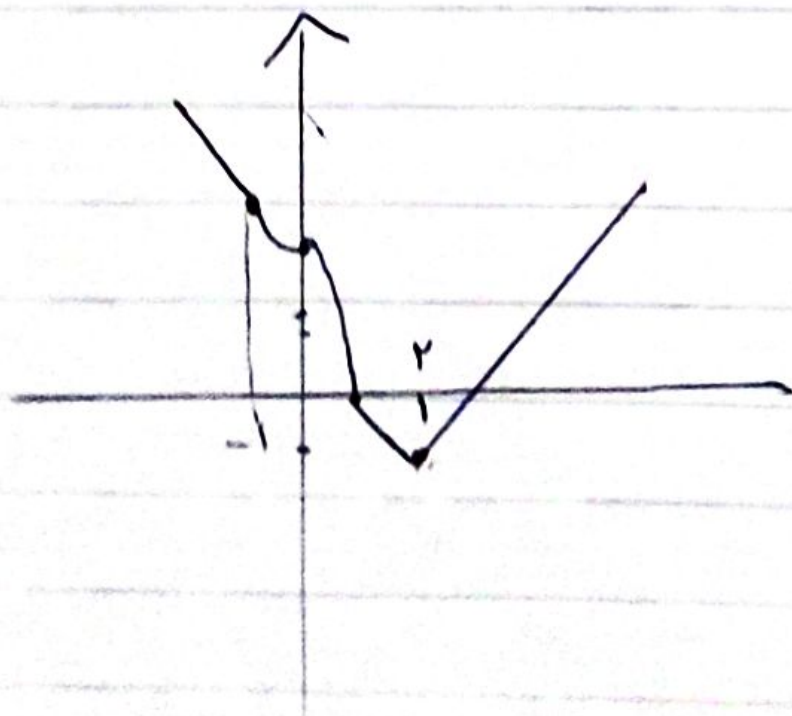
$R_f: [2, +\infty)$

5

ب) $y = |x-2| + |x-1| + |x|$

	$x < 0$	$0 \leq x < 1$	$1 \leq x < 2$	$x \geq 2$
$x-2$	-	-	-	+
$x-1$	-	-	+	+

$y = \begin{cases} -x + 2 - x + 1 + x & x < 0 \\ -x + 2 - x + 1 - x & 0 \leq x < 1 \\ -x + 2 + x - 1 - x & 1 \leq x < 2 \\ x - 2 + x - 1 - x & x \geq 2 \end{cases}$



$R_f: [-1, +\infty)$

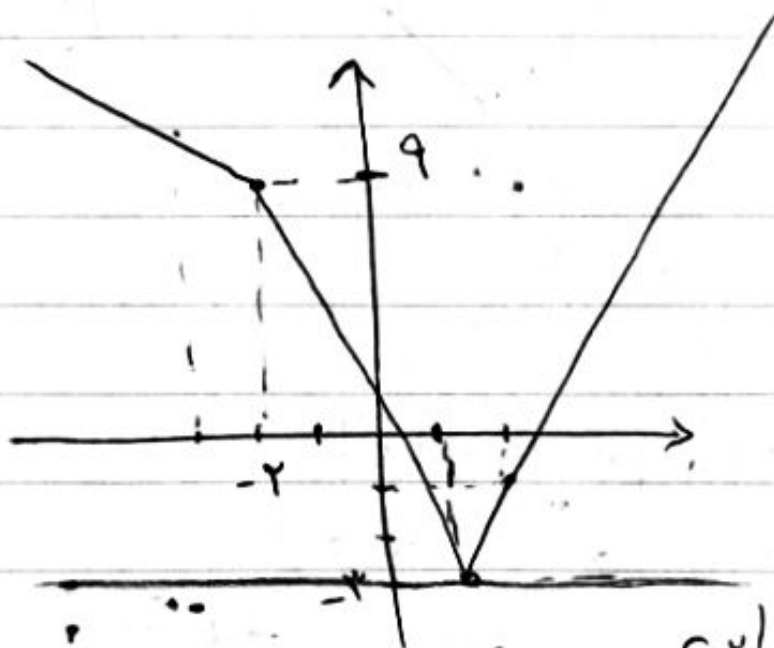
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$$|3x-3| - |x+2| = k \quad -۲.$$

$$y = |3x-3| - |x+2|$$

$\downarrow$ $\downarrow$
 $x=1$ $x=-2$

۳	-۲	۱	۲
۱۱	۹	-۳	-۱



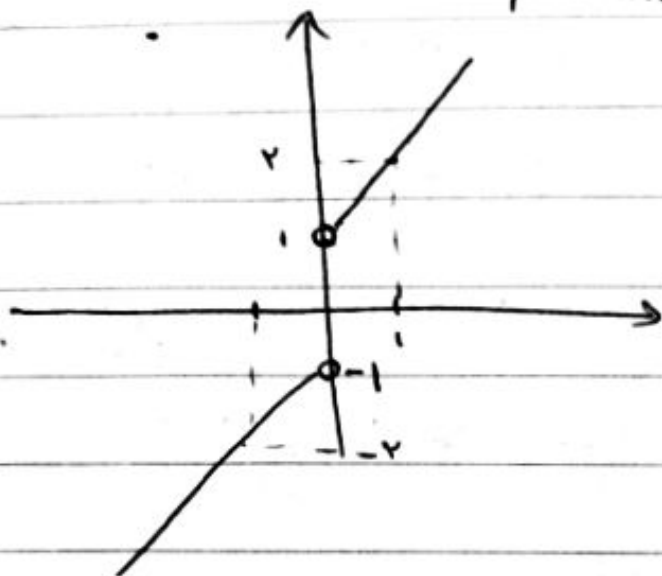
حالا باید تقاطع این
 نمودار با خط $y=k$ را بررسی کنیم
 و سه جابجایی فقط یک ریشه یعنی یک نقطه برخورد
 دارند پس $k = -۳$

جابجایی بالاتر از ۳ - دو برخورد و بیش تر از ۳ - هیچ
 برخورد نداشته

س

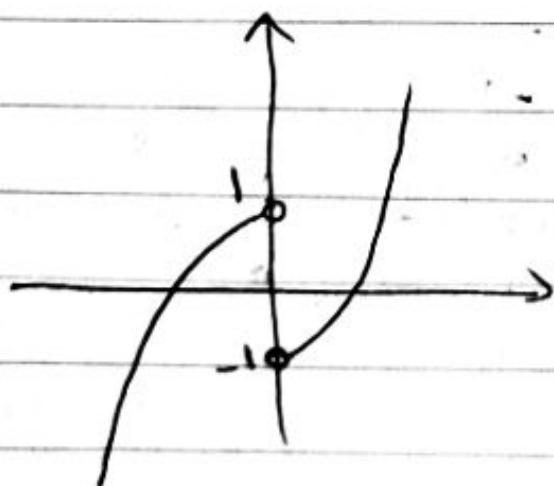
تاریخ

$$\text{الف) } y = x + \frac{x}{|x|} = \begin{cases} x + \frac{x}{x} = x + 1 & x > 0 \\ x + \frac{x}{-x} = x - 1 & x < 0 \end{cases}$$



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$$\text{ب) } y = x|x| - \frac{x}{|x|} = \begin{cases} x^2 - 1 & x > 0 \\ -x^2 + 1 & x < 0 \end{cases}$$



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الف) $y = [x] - 2x$

$-2 \leq x < -1 \rightarrow [x] = -2 \rightarrow y = -2 - 2x$

x	-2	-1
y	2	0

$-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow y = -1 - 2x$

x	-1	0
y	1	-1

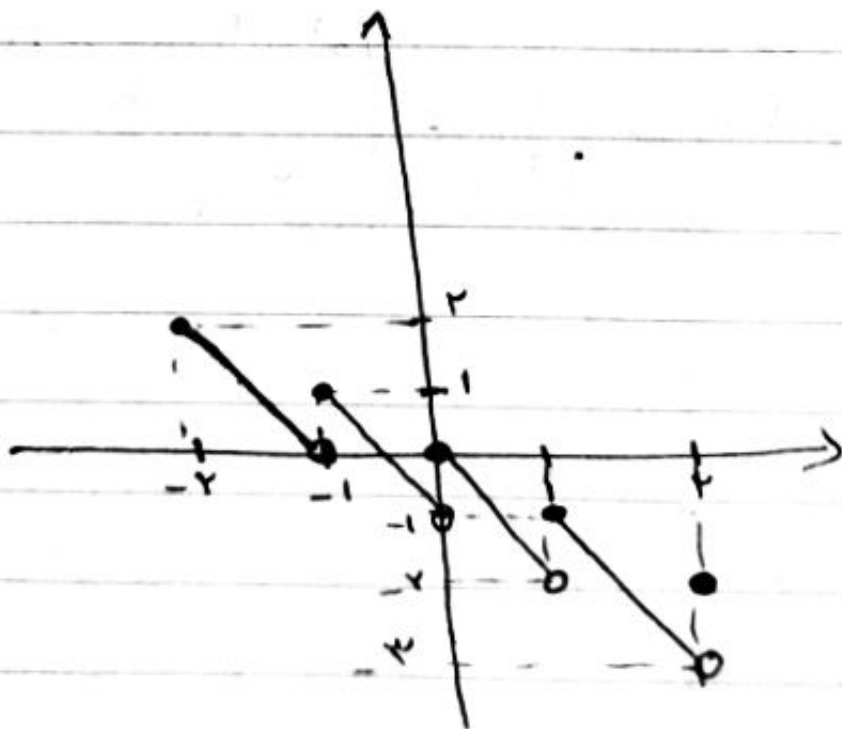
$0 \leq x < 1 \rightarrow [x] = 0 \rightarrow y = -2x$

x	0	1
y	0	-2

$1 \leq x < 2 \rightarrow [x] = 1 \rightarrow y = 1 - 2x$

x	1	2
y	-1	-3

$x = 2 \rightarrow [x] = 2 \rightarrow y = 2 - 2 = 0 \rightarrow \begin{bmatrix} 2 \\ -2 \end{bmatrix}$



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در صورت

ب) $y = [x] |x|$

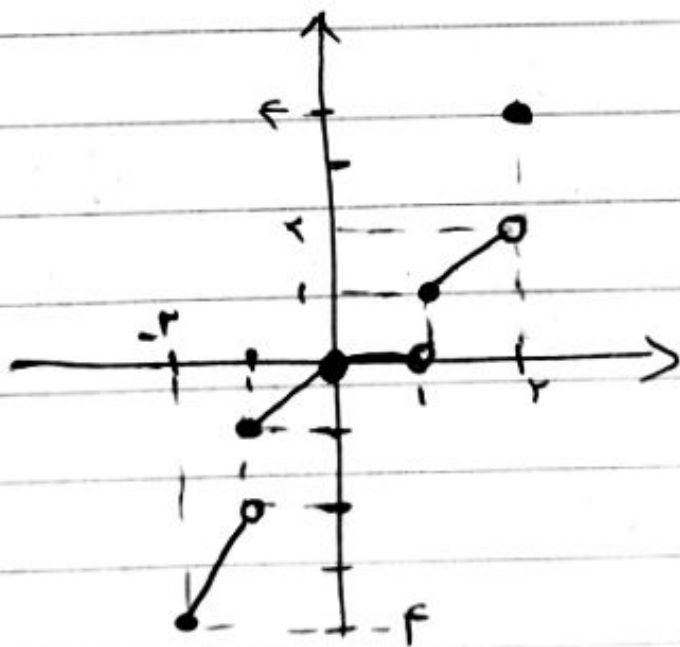
$-2 \leq x < -1 \rightarrow [x] = -2 \rightarrow y = -2|x| = 2x$ $\begin{array}{r|l} x & -2 & -1 \\ y & -4 & -2 \end{array}$

$-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow y = -1|x| = x$ $\begin{array}{r|l} x & -1 & 0 \\ y & -1 & 0 \end{array}$

$0 \leq x < 1 \rightarrow [x] = 0 \rightarrow y = 0$ $\begin{array}{r|l} x & 0 & 1 \\ y & 0 & 0 \end{array}$

$1 \leq x < 2 \rightarrow [x] = 1 \rightarrow y = |x| = x$ $\begin{array}{r|l} x & 1 & 2 \\ y & 1 & 2 \end{array}$

$x = 2 \rightarrow y = 2 \times 2 = 4 \rightarrow \begin{bmatrix} 2 \\ 4 \end{bmatrix}$





الف) $x = [x] \rightarrow R_f, [0, 1] = \mathbb{Q}$

ب) $x = [x] \rightarrow R_f, [0, 1]$



د) $R_f, [-1, 0] \rightarrow [x] = x$

الف) $-\sqrt{13} < x \sin u + x \cos u < \sqrt{13}$

$R_f, [-\sqrt{13}, \sqrt{13}]$

ب) $R_f, [-5, 5]$

ج) $-\sqrt{19} < x \sin u + x \cos u < \sqrt{19} \rightarrow R_f, [-4, 5]$

1,5

نقطة اعداد صحيح

د) $-\sqrt{5} < x \cos u - x \sin u < \sqrt{5} \rightarrow R_f, [0, \sqrt{5}]$

الف) $y = \sin^2 x + 3 \sin x + 2$

$\sin x = t \rightarrow y = t^2 + 3t + 2$

$t = -\frac{3}{2} \rightarrow y = -\frac{1}{4}$

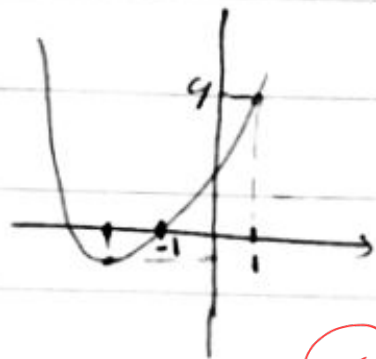
$-1 \leq t \leq 1$

$\rightarrow t = -1 \rightarrow y = 0$

$t = 1 \rightarrow y = 4$

$R_f = [0, 4]$

تابع پس
[ادامه] صعودی است پس



ب) $y = 2 \sin^2 x + 3 \sin x + 2$

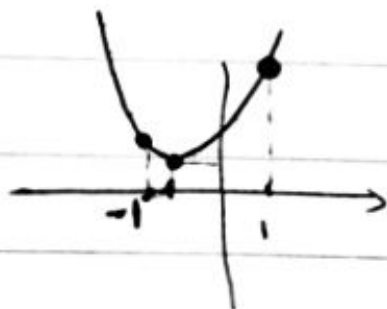
$t = \sin x \rightarrow y = 2t^2 + 3t + 2$

$t = \frac{-3}{4} \rightarrow y = \frac{5}{8}$

$-1 \leq t \leq 1$

$t = 1 \rightarrow y = 5$

$R_f = [\frac{5}{8}, 5]$



$$\text{الف) } y = \sin^2 x + f \cos^2 x \quad - \wedge$$

$$= \underbrace{\sin^2 x + \cos^2 x}_1 + f \cos^2 x = 1 + f \cos^2 x$$

$$-1 \leq \cos x \leq 1 \xrightarrow{x \rightarrow \pm \pi} -1 \leq \cos^2 x \leq 1 \xrightarrow{x \rightarrow 0} 1 \leq 1 + f \cos^2 x \leq f$$

$$R_f = [1, f]$$

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$$\text{ب) } y = \frac{f \cot x}{1 + \cot^2 x} = \frac{\frac{f \cos x}{\sin x}}{1 + \frac{\cos^2 x}{\sin^2 x}} = \frac{\frac{f \cos x}{\sin x}}{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}$$

$$= \frac{f \cos x}{\sin x} \times \frac{\sin^2 x}{1} = f \cos x \sin x = \sin^2 x$$

$$\rightarrow R_f = [-1, 1]$$

$$\text{الق) } y = \sin^4 x + \cos^4 x = (\sin^2 x)^2 + (\cos^2 x)^2 \quad -9$$

$$\text{فيل ونجول} = \underbrace{(\sin^2 x + \cos^2 x)}_1 \underbrace{(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)}_{\text{5}}$$

$$(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x$$

$$= 1 - 2(\sin^2 x \cos^2 x)^2 = 1 - 2\left(\frac{\sin^2 x}{2}\right)^2$$

$$= 1 - \frac{1}{2} (\sin^2 x)^2$$

$$0 \leq (\sin^2 x)^2 \leq 1 \xrightarrow[\text{+1}]{x = \frac{\pi}{2}} 0 \geq -\frac{1}{2} (\sin^2 x)^2 \geq -\frac{1}{2}$$

$$\rightarrow 1 \geq -\frac{1}{2} (\sin^2 x)^2 + 1 \geq \frac{1}{2} \rightarrow$$

$$R_f = \left[\frac{1}{2}, 1 \right]$$

(ب) 9

$$\frac{1}{2} \leq \sin^2 x + \cos^2 x \leq 1$$

$$\rightarrow [y_1] = R_f = \{0, 1\}$$

الف) $y = \frac{2x^2 + 7x - 4}{x + 4}$

1,0

-10

$$\begin{array}{r} 2x^2 + 7x - 4 \mid x + 4 \\ -2x^2 + 8x \\ \hline -x - 4 \\ +x + 4 \\ \hline 0 \end{array}$$

$y = 2x - 1, x \neq -4$

ریشه فرج

اگر $x = -4 \rightarrow y = 2(-4) - 1 = -9$

پس $R_f = \mathbb{R} - \{-9\}$

چون $2x - 1$ خط است.

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + x + 1)}{x-1}$

$\rightarrow y = x^2 + x + 1, x \neq 1$

اگر $x = \frac{-b}{2a} = \frac{-1}{2} \rightarrow y = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + 1 = \frac{3}{4}$

$x = 1 \rightarrow y = 1 + 1 + 1 = 3$

$\rightarrow R_f = \left[\frac{3}{4}, +\infty\right) - \{3\}$

