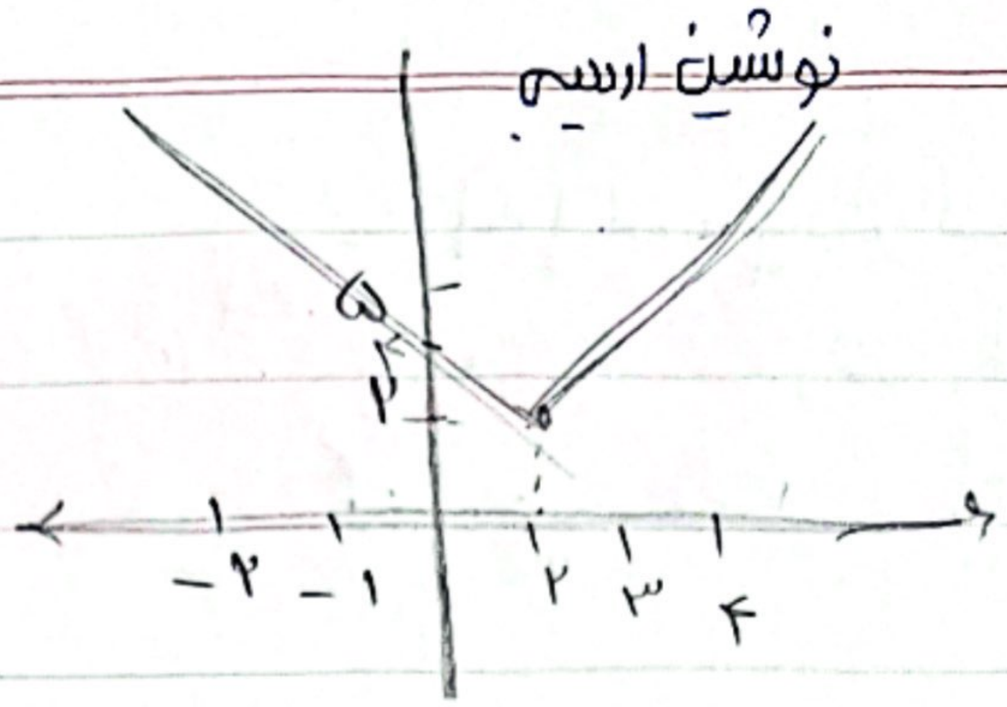
نوشین اردیبه

۱) الف) $y = x + |2x-4|$

$$y=\begin{cases} x+2x-4 & x\geq 2 \\ 2 & x=2 \\ x-2x+4 & x<2 \end{cases}$$



$$\Rightarrow \begin{cases} 3x-4 & x\geq 2 \quad R_f=[2,+\infty) \\ 2 & x=2 \quad \neq 2 \\ -x+4 & x<2 \quad R_f=[2,+\infty \end{cases} \Big\} \quad R_f=[2,+\infty)$$

ب) $y=|x-2|+|x-1| \div |x|$

$$\begin{cases} x-2+x-1-x & x\geq 2 \\ -x+2+x-1-x & x\geq 1 \\ -x+2-x+1-x & x\geq 0 \end{cases} \Rightarrow \begin{cases} x-3 & x\geq 2 \\ -x+1 & x\geq 1 \\ -3x+3 & x\geq 0 \end{cases}$$

$R_f=[-1,+\infty)$

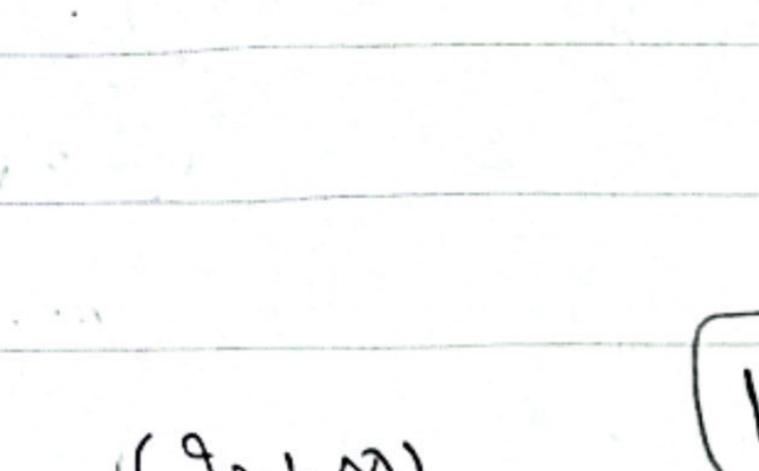
۲) $|3x-3|-|x+2|=k \quad k=?$

$$\begin{cases} -3x+3+x+2 & x<-2 \quad (9,+\infty) \\ -3x-3-x-2 & -2\leq x<1 \quad [6,-3) \\ 3x-3-x-2 & x\geq 1 \quad [-3,+\infty) \end{cases} \Rightarrow R_f=[-3,9]$$

$k=9$

۳) حل) $x+\frac{x}{|x|}=y \quad x\neq 0$

$|x|=x$



if $x>0 \quad x+\frac{x}{x}=x+1 \quad R_f=[1,+\infty)$

if $x<0 \quad x+\frac{x}{-x}=x-1 \quad R_f=(-\infty,-1]$

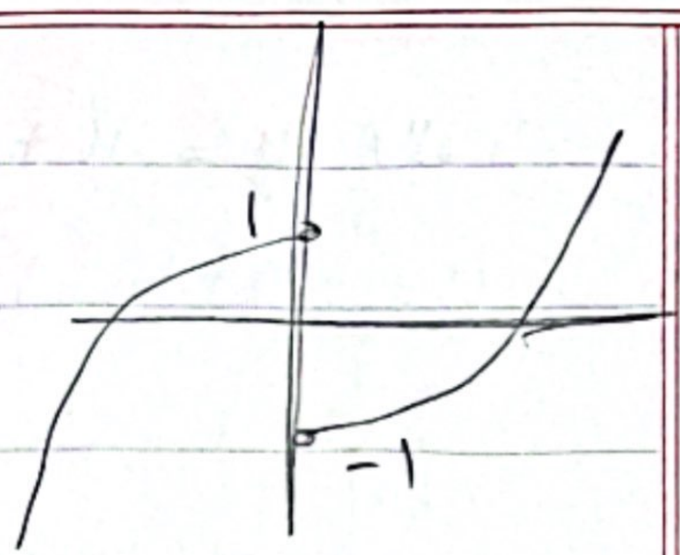
$R_f=(-\infty,-1] \cup [1,+\infty)$

Sibo

ب) $y = x|x| - \frac{x}{|x|}$

if $x>0$ x^2-1 $R_f=[0,+\infty)$

if $x<0$ $-x^2+1$ $R_f=(-\infty,0]$



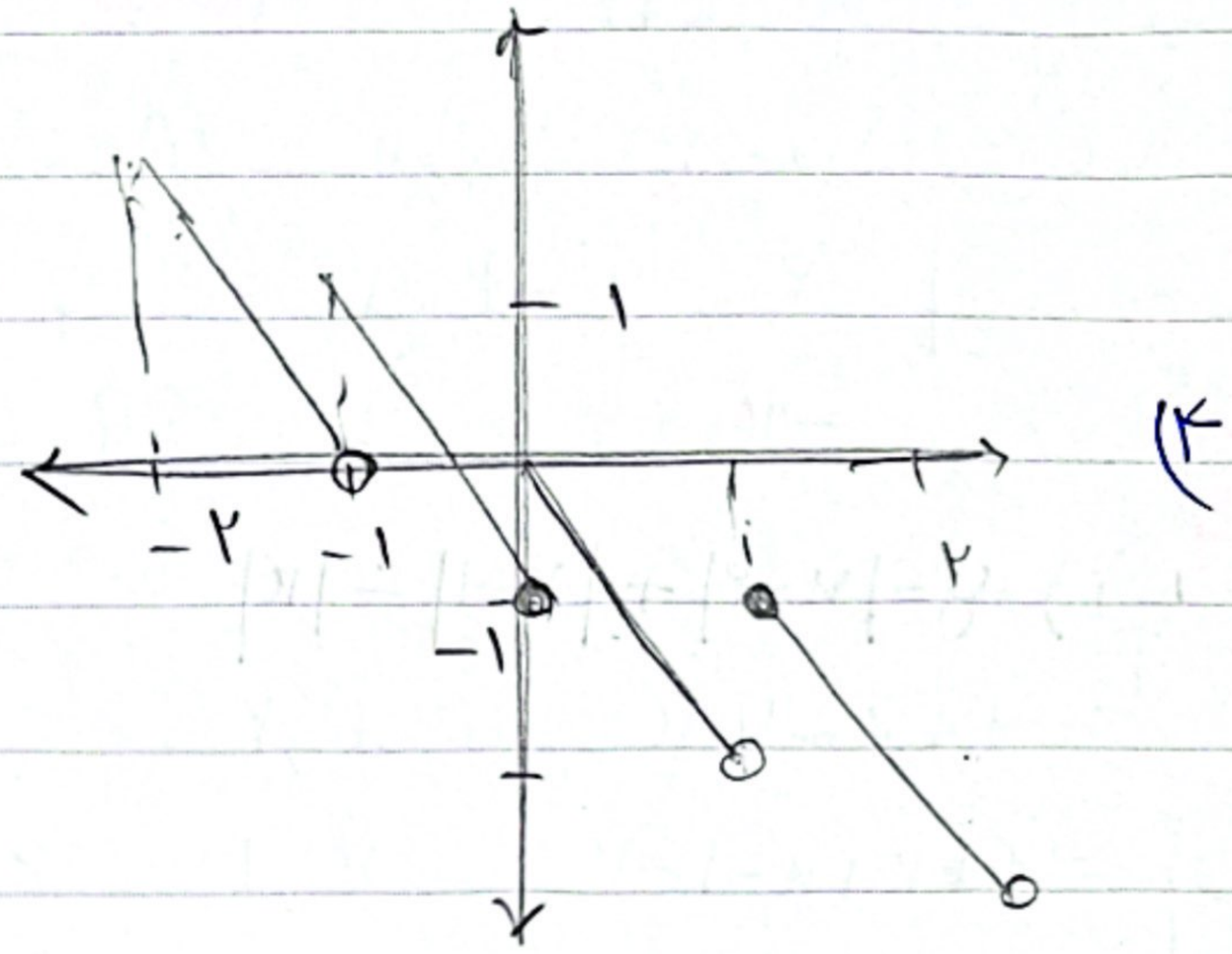
$\boxed{R_f = \mathbb{R}}$

الف) $y = [x] - 2x$

$D_f = \mathbb{R}$

$[-2,2]$ ← بازه ی

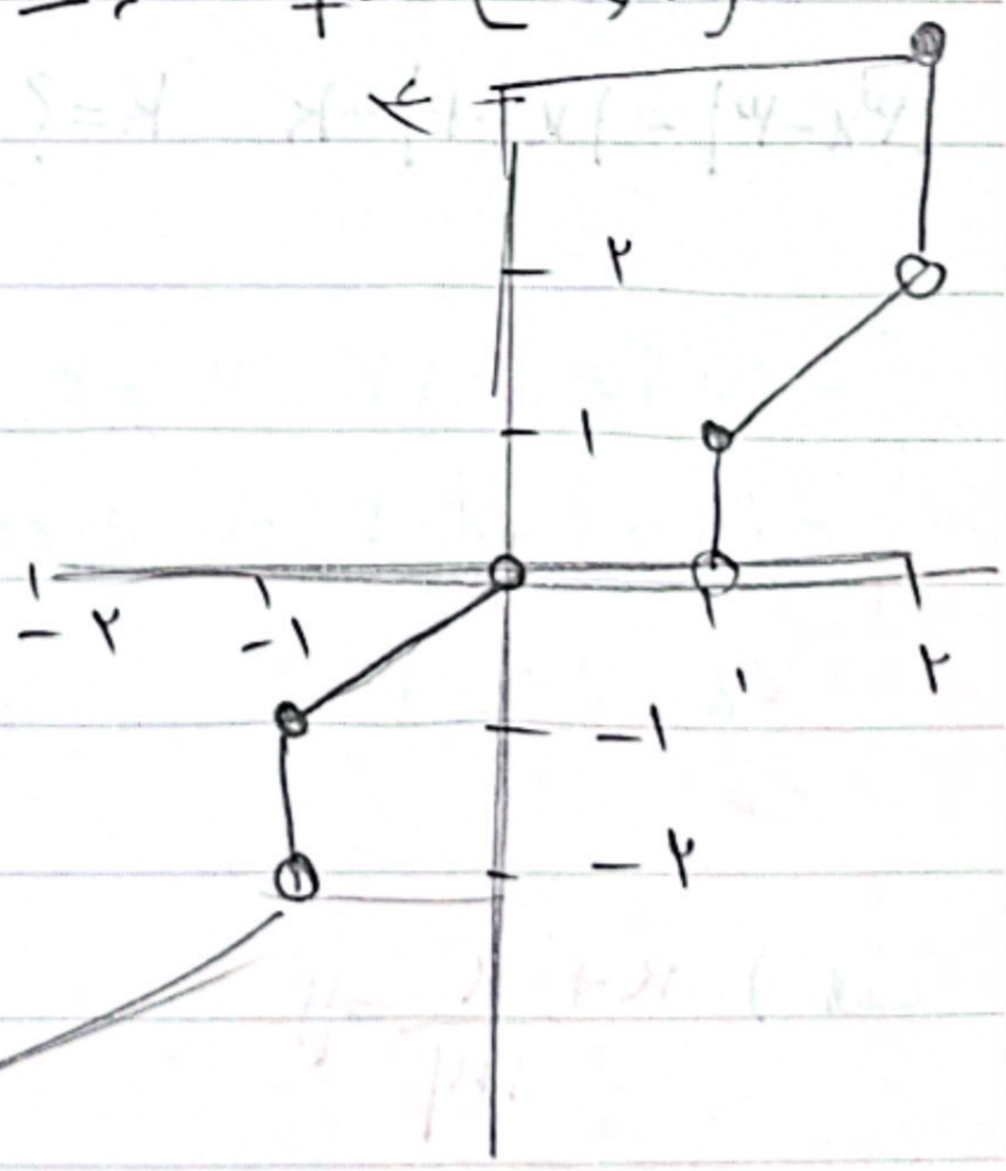
$D_f = [-2,2]$



ب) $y = [x]|x|$

$D_f = \mathbb{R} \Rightarrow$ بازه $\Rightarrow D_f = [-4,4]$

$[-4,4]$



۵) الف) $y = \pi x - [\pi x]$

$0 \leq x - [x] < 1$ طبق فرمول

$R_f = \pi x - \pi[x] = [0,1)$

ب) $y = 4x - 4[x]$. $y = 4\underbrace{(x - [x])}_{[0,1)}$ $R_f = [0,4)$

ج) $y = x - 5\left[\frac{x}{5}\right]$

$R_f = [0,5)$

د) $y = [2x] - 2x$

$[2x] \leq 2x$ پس $R_f = (-1,0]$

۶) الف) $y = 3\sin x + 2\cos x$

$-\sqrt{3^2+2^2} \leq 3\sin x + 2\cos x \leq \sqrt{3^2+2^2}$

$R = [-\sqrt{13}, +\sqrt{13}]$

ب) $y = 4\sin x - 3\cos x$

$-1 < \sin x, \cos x < 1$ $R_f = [-5,5]$

$\sin^2 x + \cos^2 x = 1$

ج) $y=[2\sin x+3\cos x]$

$-\sqrt{2^2+3^2} \le 2\sin x+3\cos x \le \sqrt{2^2+3^2}$

$-\sqrt{13} \le 2\sin x+3\cos x \le \sqrt{13}$

$3<\sqrt{13}<4$

$-3>-\sqrt{13}>-4$

$R_f=[-\sqrt{13},+\sqrt{13}]=\{-3,-2,-1,0,1,2,3,4\}$

1	-3
2	-2
3	-1
4	0
5	

ا) $y=\sqrt{\cos x-2\sin x}$

$\cos x-2\sin x \ge 0 \quad \frac{\cos x}{\sin x} \ge 2 \quad \sin x \neq 0$

$R_f=\sqrt{[-\sqrt{5},\sqrt{5}]}=[0,\sqrt{5}]$

ضریب فرد

ه) $y=5\sin^2 x+2\sin x+2$ (۷

$\sin=1 \quad y=9$

$\sin=-1 \quad y=0 \qquad R=[0,9]$

$\sin=-\frac{b}{2a}=-\frac{3}{2} \longrightarrow \sin x \neq -\frac{3}{2}$

ضریب زوج

ب) $y=6\sin^2 x+3\sin x+2$

$\sin=0 \quad y=2$

$\sin=1 \quad y=7 \qquad R=[\frac{14}{8},7]$

$\sin=-\frac{b}{2a}=-\frac{3}{4} \quad y=\frac{14}{8}$

۹ الف

الف) $y=\sin^4 x+\cos^4 x$

$R=(\frac{1}{2},1]$

$1-2 < \sin^{2k}+\cos^{2k} \le 1$

$1-2 \quad \frac{1}{2} < \sin^4 x+\cos^4 x \le 1$

ب) $y = (\sin^2 x + \cos^2 x)^{2k-1}$

$-1 \leq \sin^{2k} x + \cos^{2k} x \leq 1$

$\frac{1}{2} \leq \sin^2 x + \cos^2 x \leq 1$

بنابراین $R[\sin^2 x + \cos^2 x] = \{0, 1\}$

۸) الف) $y = \sin^2 x + 4\cos^2 x$

$y = \underbrace{\sin^2 x + \cos^2 x}_{1} + 3\cos^2 x = [1, 4) = R$

$1 + 3\underbrace{\cos^2 x}_{[0,1)}$

ب) $y = \frac{2\cot x}{1 + \cot^2 x} = \frac{2\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = \frac{\sin^2 x \, 2\cos x}{\sin x} =$

$\sin x \, 2\cos x$

۱۰) الف) $y = \frac{2x^2 + 7x - 4}{x + 4}$ $\quad y = 2x - 1 \quad ; \quad x \neq -4$

$R_f = \mathbb{R}$

ب) $y = \frac{(x^3 - 1)}{x - 1} = \frac{(x-1)(x^2 + x + 1)}{x - 1}$ ، $\left(-\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{3}{4}$

$-\frac{b}{2a} = -\frac{1}{2}$ $\quad R_f = [\frac{3}{4}, +\infty)$