

الف) $y = x + |2x - 4|$ $x=2$

$$\begin{cases} x \geq 2 & 3x - 4 \rightarrow R = [2, +\infty) \\ x < 2 & -x + 4 \rightarrow R = (-\infty, 2) \end{cases} \cup \rightarrow [2, +\infty)$$

ب) $y = |x-2| + |x-1| - |x|$

$$\begin{array}{l|l|l|l} x-2 & x-1 & -x & \\ \hline x \geq 2 & x-1 & -x & x-2+x-1-x = x-3 \\ 2 > x & x-1 & -x & x-2-x-1-x = -2x-3 \\ x < 2 & x-1 & -x & x-2-x-1-x = -2x-3 \end{array}$$

$R = (-\infty, 2)$ $R = [2, 3]$ $R = [-1, 0]$

• $\cup \rightarrow (-\infty, 2) \cup [2, 3] \cup [-1, 0] = (-\infty, 3]$

$|3x-3| - |x+2| = k$

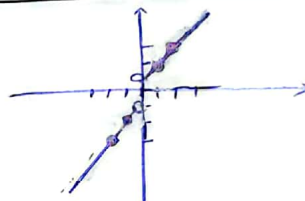
$$\begin{array}{l|l|l} 3x-3 & x+2 & \\ \hline x \geq -2 & x+2 & 3x-3-x-2 = 2x-5 \\ x < -2 & -x-2 & 3x-3-x-2 = 2x-5 \end{array}$$

$R = (-\infty, 9)$ $R = [-3, 9]$ $R = [-3, +\infty)$

ریشه داریم چون $x = -2 \rightarrow k = 9$
 در این نقاطی از تقاطعها منفرد است. $x = 1 \rightarrow k = -3$

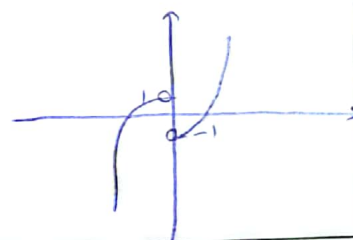
الف) $y = x + \frac{x}{|x|}$ $x \neq 0$

$$\begin{cases} x > 0 \rightarrow x+1 \\ x < 0 \rightarrow x-1 \end{cases}$$



ب) $y = x|x| - \frac{x}{|x|}$ $x \neq 0$

$$\begin{cases} x > 0 \rightarrow x^2 - 1 \\ x < 0 \rightarrow -x^2 + 1 \end{cases}$$

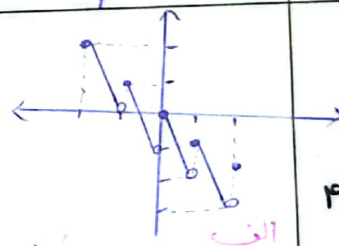
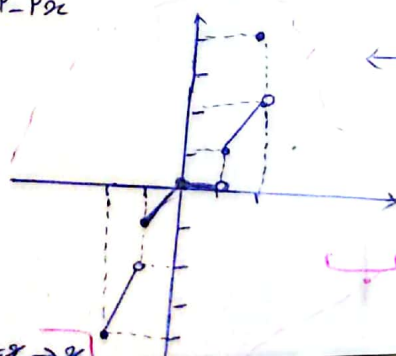


الف) $y = [x] - 2x$ $[-2, -1) \rightarrow [x] = -2 \rightarrow -2 - 2x$

$[-1, 0) \rightarrow [x] = -1 \rightarrow -1 - 2x$

$[0, 1) \rightarrow [x] = 0 \rightarrow -2x$

$[1, 2) \rightarrow [x] = 1 \rightarrow 1 - 2x$



ب) $[x] |x|$

$[-2, -1) \rightarrow [x] = -2, |x| = -x \rightarrow 2x$ $-2 \rightarrow -4$

$[-1, 0) \rightarrow [x] = -1, |x| = -x \rightarrow x$

$[0, 1) \rightarrow [x] = 0, |x| = x \rightarrow x$

$[1, 2) \rightarrow [x] = 1, |x| = x \rightarrow x$

الف) $y = 3x - [3x] \rightarrow 0 \leq 3x - [3x] < 1 \rightarrow R = [0, 1)$

ب) $y = 4x - 4[x] \rightarrow 4(x - [x]) \rightarrow R = [0, 4)$

ج) $y = x - 5[\frac{x}{5}] = 5(\frac{x}{5} - [\frac{x}{5}]) \rightarrow R = [0, 5)$

د) $y = [2x] - 2x \rightarrow R = [-1, 0]$

$0 \leq \heartsuit - [\heartsuit] < 1$
 $-1 < [\heartsuit - \heartsuit] \leq 0$

الف) $y = 3\sin x + 4\cos x \rightarrow -\sqrt{4+9} \leq 3\sin x + 4\cos x \leq \sqrt{4+9}$ $-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

$R = [-\sqrt{17}, \sqrt{17}]$

ب) $y = 4\sin x - 3\cos x \rightarrow -5 \leq 4\sin x - 3\cos x \leq 5$ $R = [-5, 5]$

ج) $y = [r\sin x + 5\cos x] \rightarrow -\sqrt{r^2+25} \leq r\sin x + 5\cos x \leq \sqrt{r^2+25}$

$R = [-\sqrt{r^2+25}, \sqrt{r^2+25}]$ $\rightarrow -\sqrt{25} \leq \cos x - 3\sin x \leq \sqrt{25}$
 $\sqrt{25} \leq \cos x - 3\sin x \leq \sqrt{25}$

الف) $y = \sin^2 x + 3\sin x + 4$

$\min \sin = -1 \rightarrow 1 - 3 + 4 = 2 \rightarrow \min$
 $\max \sin = 1 \rightarrow 1 + 3 + 4 = 8 \rightarrow \max$
 $y = \frac{9}{4} - \frac{9}{4} + 4 = 4$

$R = [2, 8]$

$R = [2, 4]$

ب) $y = \sin^2 x + 3\sin x + 4$

$\min \sin = -1 \rightarrow 1 - 3 + 4 = 2$
 $\max \sin = 1 \rightarrow 1 + 3 + 4 = 8$
 $y = \frac{9}{4} - \frac{9}{4} + 4 = 4$

$R = [\frac{9}{4}, 4]$

الف) $\sin^2 x + 4\cos^2 x = 1 - \cos^2 x + 4\cos^2 x = 3\cos^2 x + 1 \rightarrow 0 \leq \cos^2 x \leq 1$
 $0 \leq 3\cos^2 x \leq 3$
 $1 \leq 1 + 3\cos^2 x \leq 4$
 $R = [1, 4]$

ب) $y = \frac{4\cos^2 x}{1 + \cos^2 x} = \frac{4 \frac{\cos^2 x}{\sin^2 x}}{1 + \frac{\cos^2 x}{\sin^2 x}} = 4 \cos^2 x \sin^2 x = \sin^2 x \rightarrow -1 \leq \sin^2 x \leq 1$
 $R = [-1, 1]$

الف) $y = \sin^4 x + \cos^4 x \xrightarrow{k=3} 1 - 3 \leq \sin^4 x + \cos^4 x \leq 1$ $1 - k \leq \sin^k x + \cos^k x \leq 1$
 $\frac{1}{4} \leq \sin^4 x + \cos^4 x \leq 1$ $R = [\frac{1}{4}, 1]$

ب) $y = [\sin^4 x + \cos^4 x] \xrightarrow{k=4} 1 - 4 \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow \frac{1}{4} \leq \sin^4 x + \cos^4 x \leq 1$
 $R = [\frac{1}{4}, 1] = \{0, 1\}$

الف) $y = \frac{x^2 + 4x - 4}{x + 4} \rightarrow x^2 + 4x - 4 = \frac{(x+4)^2 + 4(x+4) - 16}{x+4} = \frac{(x+4)(x-1)}{x+4} = x-1$

$(x+4)(x-1) \rightarrow y = \frac{(x+4)(x-1)}{x+4} = x-1 \xrightarrow{x=-4} y = -5$ $R = R - \{-5\}$

ب) $y = \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{x-1} = x+1$

$x=1 \rightarrow y=2$

$R = [\frac{5}{4}, +\infty) - \{2\}$

$\rightarrow -\frac{b}{a} = -\frac{1}{4} \rightarrow y = \frac{1}{4} - \frac{1}{4} + 1 = \frac{1}{2}$
 $[\frac{1}{4}, +\infty) \rightarrow \frac{1}{2}$