Subject ۱۴۰۴/۶/۳۰
Date

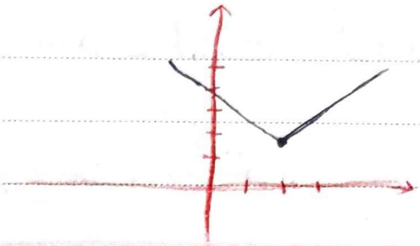
به نام خدا

فاطمه متقی - ۷

۱)

الف) $y = x + |2x-4|$

	2	
$x-2x+4$		$x+2x-4$
$-x+4$		$3x-4$

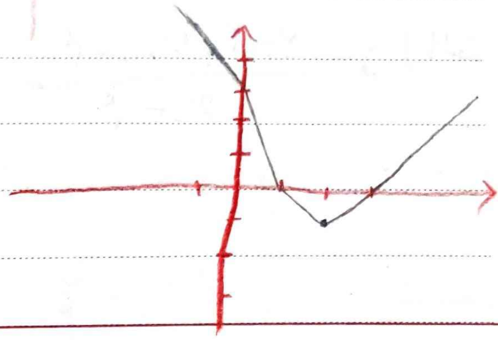


$R_f = [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$

	0		1		2	
$-x+2-x+1+x$		$-x+2-x$		$-x+2+x-1$		$x-2+x-1-x$
$-x+3$		$-x+1$		$-x$		$x-3$
		$-3x+3$		$-x+1$		

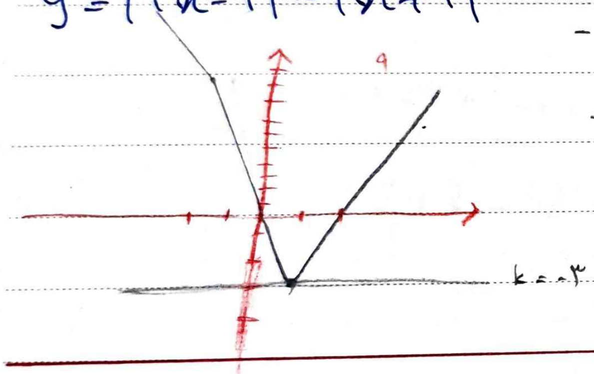
$R_f = [-1, +\infty)$



۲)

$y = |3x-3| - |x+2|$

	-2		1	
$-3x+3+x+2$		$-3x+3-x-2$		$3x-3-x-2$
$-2x+5$		$-4x+1$		$2x-5$



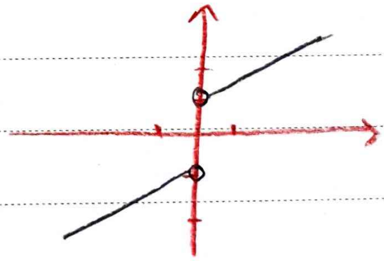
$k = -3$

برای اینکه فقط یک ریشه ← فقط در یک نقطه برابر k

۳)

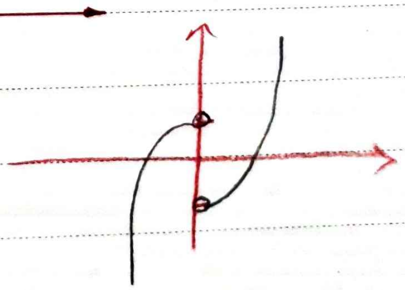
الف) $y = x + \frac{x}{|x|}$

	0	
$x-1$		$x+1$



ب) $y = x|x| - \frac{x}{|x|}$

	0	
$-x^2+1$		x^2-1



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۴)

الف) $y = [x] - 2x$

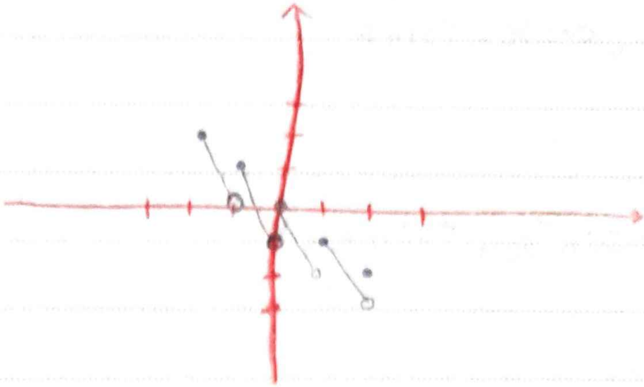
$-2 \le x < -1 \rightarrow -2x - 2$

$-1 \le x < 0 \rightarrow -2x - 1$

$0 \le x < 1 \rightarrow -2x$

$1 \le x < 2 \rightarrow 1 - 2x$

$x = 2 \rightarrow -2$



ب) $y = [x]\,|x|$

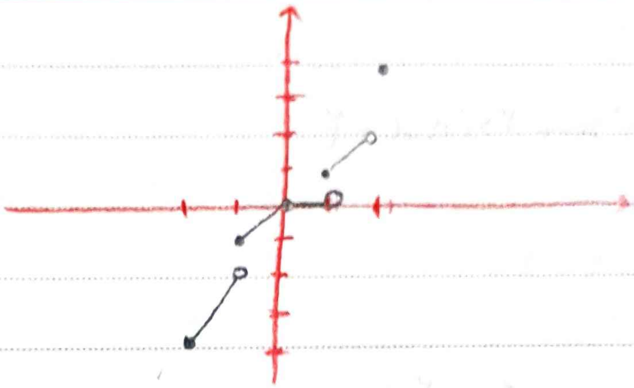
$-2 \le x < -1 \rightarrow 2x$

$-1 \le x < 0 \rightarrow x$

$0 \le x < 1 \rightarrow 0$

$1 \le x < 2 \rightarrow x$

$x = 2 \rightarrow 4$



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۵) $0 \le x - [x] < 1$

الف) $y = 3x - [3x] \rightarrow R_f = [0, 1)$

ب) $y = 4x - 4[x] \rightarrow 4(x - [x]) \Rightarrow R_f = [0, 4]$

ج) $y = x - 5\left[\frac{x}{5}\right] \rightarrow 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right) \Rightarrow R_f = [0, 5]$

د) $y = [2x] - 2x \rightarrow -(2x - [2x]) \Rightarrow R_f = (-1, 0]$

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۶)

الف) $y = 3\sin x + 2\cos x \rightarrow -\sqrt{3^2 + 2^2} \le 3\sin x + 2\cos x \le \sqrt{3^2 + 2^2}$

$\hookrightarrow R_f = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin x - 3\cos x \rightarrow -\sqrt{4^2 + (-3)^2} \le 4\sin x - 3\cos x \le \sqrt{4^2 + (-3)^2}$

$\hookrightarrow -\sqrt{25} \le 4\sin x - 3\cos x \le \sqrt{25} \Rightarrow R_f = [-5, 5]$

ج) $y = [2\sin x + 5\cos x] \rightarrow -\sqrt{2^2 + 5^2} \le 2\sin x + 5\cos x \le \sqrt{\underbrace{2^2 + 5^2}_{29}}$

$-6 \le [2\sin x + 5\cos x] \le 5 \qquad R_f = [-6, 5]$

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ب) $y=\sqrt{\cos x - 2\sin x} \rightarrow -\sqrt{1^2+(-2)^2} \le \cos x - 2\sin x \le \sqrt{1^2+(-2)^2} = \sqrt{5}$

$0 \le \sqrt{\cos x - 2\sin x} \le \sqrt[4]{5} \Rightarrow R_f=[0, \sqrt[4]{5}]$

7) الف) $y=\sin^2 x+3\sin x+2$ $\qquad R_f=[0,6]$

↳ $\sin x=-1 \rightarrow 1-3+2=0$

↳ $\sin x=1 \rightarrow 1+3+2=6$

↳ $\sin x=\frac{-b}{a}=-3 \rightarrow 9-9+2=2$

ب) $y=2\sin^2 x+3\sin x+2$ $\qquad R_f=[1,7]$

↳ $\sin x=-1 \rightarrow 2-3+2=1$ $\qquad$ ↳ $\sin x=\frac{-b}{a}=\frac{-3}{2} \rightarrow 2\times\frac{9}{4}-\frac{9}{2}+2=2$

↳ $\sin x=1 \rightarrow 2+3+2=7$

8) الف) $y=\sin^2 x+4\cos^2 x \Rightarrow \sin^2 x+4(1-\sin^2 x)$ $\qquad \boxed{R_f=[1,4]}$

↳ $\sin^2 x-4\sin^2 x+4=-3\sin^2 x+4 \Rightarrow 0\le \sin^2 x\le 1$

$\underline{\times(-3)}$ $\quad -3\le -3\sin^2 x\le 0 \xrightarrow{+4} 1\le -3\sin^2 x+4\le 4$

ب) $y=\frac{2\cot x}{1+\cot^2 x} \rightarrow \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x\cos x = \sin 2x$

$-1\le \sin 2x\le 1 \rightarrow R_f=[-1,1]$

9) $2^{1-k}\le \sin^{2k} x+\cos^{2k} x\le 1$

الف) $y=\sin^6 x+\cos^6 x$

$2^{1-3}\le \sin^6 x+\cos^6 x\le 1 \Rightarrow R_f=[\frac{1}{4},1]$

ب) $[\sin^8 x+\cos^8 x] \Rightarrow \underbrace{2^{1-4}}_{\frac{1}{8}}\le \sin^8 x+\cos^8 x\le 1$

داخل براکت → برود $\qquad R_f=\{0,1\}$

PAPCO

۱۰)

الف) $y = \dfrac{2x^2 + 7x - 4}{x + 4} = 2x - 1$

$$\begin{array}{r|l} 2x^2 + 7x - 4 & x + 4 \\ \cline{2-2} -2x^2 - 8x & 2x - 1 \\ \hline -x - 4 & \\ +x + 4 & \\ \hline 0 & \end{array}$$

$\xrightarrow{x \neq -4} y \neq 2(-4) - 1 = -9$

$\rightarrow R_f = \mathbb{R} - \{-9\}$

ب) $y = \dfrac{x^3 - 1}{x - 1} \sim \dfrac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{x-1}} \Rightarrow x^2 + x + 1$

$\xrightarrow{x \neq 1} y \neq 1 + 1 + 1 = 3 \longrightarrow R_f = \mathbb{R} - \{3\}$