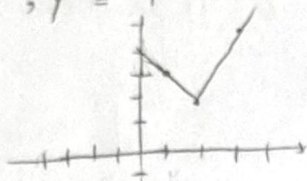


۱) $y = x + |x - 2|$

$x - 2 = 0 \rightarrow x = 2, y = 2$

$x = 1, y = 1$

$x = 3, y = 4$



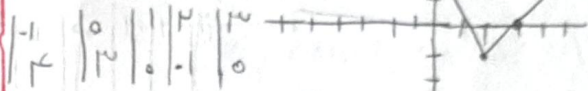
$R_f = [2, +\infty)$

۲) $|x - 2| + |x - 1| = |x|$

$x - 2 = 0 \rightarrow x = 2, y = -1$

$x - 1 = 0 \rightarrow x = 1, y = 0$

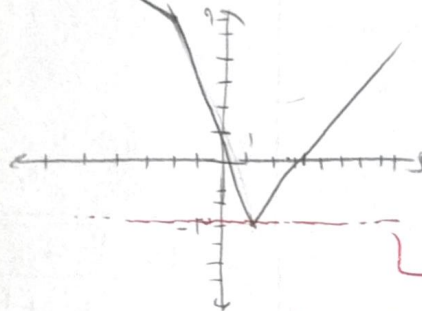
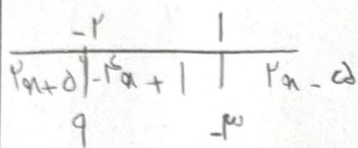
$x = 0, y = 3$



$R_f = [-1, +\infty)$

۳) $|x - 3| - |x + 2| = k$

$x + 2 = 0 \rightarrow x = -2, y = 5$, $x - 3 = 0 \rightarrow x = 3, y = -1$



$k = -1$

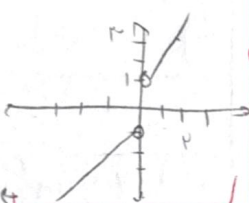
برای این یک ریشه داشته باشیم فقط
یکبار مقدار را می‌توانیم که در نقطه $y = -1$
مقدار فقط یکبار می‌شود

۴) $y = x + \frac{x}{|x|} \rightarrow x > 0 = x + 1 \rightarrow y = x + 1$

$x < 0 = x - 1 \rightarrow y = x - 1$

$x + 1 = y \rightarrow x = 2, y = 3$

$x - 1 = y \rightarrow x = -1, y = -2$



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۵) $y = x|x| - \frac{x}{|x|}$

$x > 0 \rightarrow x^2 - 1 \rightarrow x = 0, y = -1$

$x < 0 \rightarrow -x^2 + 1 \rightarrow x = 0, y = 1$



۶) $y = [x] - 2x$

$0 \leq x < 1 \rightarrow [x] = 0, 1 \leq x < 2 \rightarrow [x] = 1$

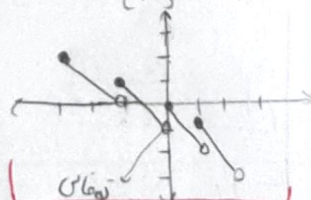
$-1 \leq x < 0 \rightarrow [x] = -1, 2 \leq x < 3 \rightarrow [x] = 2$

$-2 \leq x < -1 \rightarrow -2x$

$1 \leq x < 2 \rightarrow -2x + 1$

$-1 \leq x < 0 \rightarrow -2x - 1$

$2 \leq x < 3 \rightarrow -2x - 2$



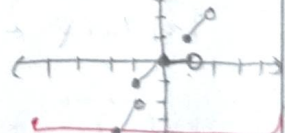
۷) $y = [x]|x| - 2 \leq x < 1 \rightarrow [x] = -2, y = -2|x|$

$0 \leq x < 1 \rightarrow [x] = 0 \rightarrow y = 0$

$1 \leq x < 2 \rightarrow [x] = 1 \rightarrow y = |x|$

$-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow y = -|x|$

$2 \leq x < 3 \rightarrow [x] = 2$



۸) $y = 2x - [2x] = 0 \leq y - [y] < 1$

$y = 2x \rightarrow 0 \leq 2x - [2x] < 1$

$R_f = [0, 1)$

۹) $y = 2x - f[x], f(x - [x])$

$0 \leq x - [x] < 1 \rightarrow 0 \leq f(x - [x]) < 1$

$R_f = [0, 1)$

۱۰) $y = x - \frac{x}{2} = \frac{x}{2} \rightarrow \frac{x}{2} - [\frac{x}{2}] < 1 \rightarrow R_f = [0, 2)$

۱۱) $y = [2x] - 2x = -(2x - [2x])$

$0 \leq 2x - [2x] < 1 \rightarrow R_f = [-1, 0]$

1) $y = r \sin \alpha + r \cos \alpha$ $-\sqrt{r^2+r^2} \leq r \sin \alpha + r \cos \alpha \leq \sqrt{r^2+r^2}$ $R_f = [-\sqrt{1^2}, \sqrt{1^2}]$	2) $y = r \sin \alpha - r \cos \alpha$ $-\sqrt{1^2+1^2} \leq r \sin \alpha - r \cos \alpha \leq \sqrt{1^2+1^2}$ $R_f = [-\sqrt{2}, \sqrt{2}]$	6
3) $y = [r \sin \alpha + 0 \cos \alpha]$ $-\sqrt{r^2+0^2} \leq r \sin \alpha + 0 \cos \alpha \leq \sqrt{r^2+0^2} \rightarrow [-\sqrt{1^2}, \sqrt{1^2}]$ $= R_f = [-1, 1]$	4) $y = \sqrt{\cos \alpha - r \sin \alpha}$ $-\sqrt{1^2+1^2} \leq \cos \alpha - r \sin \alpha \leq \sqrt{1^2+1^2} \rightarrow [-\sqrt{2}, \sqrt{2}]$ $R_f = [-\sqrt{2}, \sqrt{2}]$	7
5) $y = \sin^2 \alpha + r \sin \alpha + r$ $\max \rightarrow \sin \alpha = 1 \rightarrow 1 + r + r = 4$ $\min \rightarrow \sin \alpha = -1 \rightarrow 1 - r + r = 0$ $R_f = [0, 4]$	6) $y = r \sin^2 \alpha + r \sin \alpha + r$ $\max \rightarrow \sin \alpha = 1 \rightarrow r + r + r = 3$ $\min \rightarrow \sin \alpha = -1 \rightarrow r - r + r = 1$ $R_f = [1, 3]$	Y
7) $y = \sin^2 \alpha + r \cos^2 \alpha$ $\cos^2 \alpha = 1 - \sin^2 \alpha \rightarrow \sin^2 \alpha - r \sin^2 \alpha + r$ $-r \sin^2 \alpha + r \rightarrow \min \sin^2 \alpha = 0 \rightarrow r$ $\max \sin^2 \alpha = 1 \rightarrow 1$ $R_f = [r, 1]$	8) $y = \frac{r \cot \alpha}{1 + \cot^2 \alpha} = \frac{r \cos \alpha}{\sin \alpha} \cdot \frac{1}{1 + \cot^2 \alpha}$ $\frac{r \sin \alpha \cos \alpha}{\sin^2 \alpha + \cos^2 \alpha} = \sin \alpha \cos \alpha \rightarrow [-1, 1]$ $R_f = [-1, 1]$	A
9) $y = \sin^4 \alpha + \cos^4 \alpha \rightarrow r k \leq 4 \rightarrow k \leq r$ $r^{1-r} \leq \sin^4 \alpha + \cos^4 \alpha \leq 1 \rightarrow \frac{1}{r}, 1$ $R_f = [\frac{1}{r}, 1]$	10) $y = [\sin^4 \alpha + \cos^4 \alpha] r k \leq 1 \rightarrow k \leq r$ $r^{1-r} \leq \sin^4 \alpha + \cos^4 \alpha \leq 1 \rightarrow [\frac{1}{r}, 1]$ $R_f = [\frac{1}{r}, 1]$	9
11) $y = r \alpha^2 + u \alpha - r$ $r \alpha^2 + u \alpha - r \rightarrow \alpha^2 + u \alpha - 1 = (\alpha + \frac{u}{2})^2 - \frac{u^2}{4} - 1$ $(\alpha + \frac{u}{2})^2 - \frac{u^2}{4} - 1 = y$ $R_f = \mathbb{R} - (-\frac{u^2}{4} - 1)$	12) $y = \frac{\alpha^2 - 1}{\alpha - 1} = \frac{(\alpha - 1)(\alpha + 1 + \alpha)}{\alpha - 1}$ $\alpha^2 + 1 + \alpha = y$ $\Delta < 0 \rightarrow \frac{-1 \pm \sqrt{1 - 4(y-1)}}{2} = \frac{-1 \pm \sqrt{5 - 4y}}{2}$ $R_f = [\frac{5}{4}, +\infty)$	10