

الف) $y = x + |2x - 4|$

$y = x + (2x - 4) = 3x - 4$

اگر $2x - 4 < 0 \rightarrow x < 2$

$y = x - (2x - 4) = -x + 4$

برای $x = 2$ و $y \in (2, +\infty) \leftarrow x > 2$

برای $x < 2$ و $y \in (2, +\infty) \leftarrow y = -x + 4$

$y \rightarrow +\infty$ و $x \rightarrow +\infty$ و $y = 1$ و $x = 2$

$Ry = [2, +\infty)$

ب) $y = |x - 2| + |x - 1| - |x|$

$x = 0, 1, 2$

$(-\infty, 0), [0, 1), [1, 2), [2, +\infty)$

اگر $x < 0$ $y = 2 - x + (x - 1) - (-x) = 3 - x$ $y \in (3, +\infty)$

اگر $0 \leq x < 1$ $y = (2 - x) + (1 - x) - x = 3 - 3x$ $y \in [3, 0]$

اگر $1 \leq x < 2$ $y = (2 - x) + (x - 1) - x = 1 - x$ $y \in (-1, 0]$

اگر $x \geq 2$ $y = (x - 2) + (x - 1) - x = x - 3$ $y \in [-1, +\infty)$

$(-1, 0] \cup (0, 2] \cup (2, +\infty) \cup [-1, +\infty)$

$Ry = [-1, +\infty)$

۱) $|3x - 3| - |x + 2| = k$

$3x - 3 = 0 \rightarrow x = 1$

$x + 2 = 0 \rightarrow x = -2$

۱) $x < -2 \rightarrow -(3x - 3) - (x + 2) = k \Rightarrow -3x + 3 - x - 2 = k \Rightarrow -4x + 1 = k \Rightarrow x = \frac{1 - k}{4}$

$\frac{1 - k}{4} < -2 \Rightarrow 1 - k < -8 \Rightarrow k < 9$

۲) $-2 \leq x < 1 \rightarrow -(3x - 3) - (x + 2) = k \Rightarrow k = -3x + 3 - x - 2 \Rightarrow k - 1 = -4x \Rightarrow x = \frac{1 - k}{4}$

$-2 \leq \frac{1 - k}{4} < 1 \Rightarrow 1 - k \geq -8 \Rightarrow k \leq 9$

$\frac{1 - k}{4} < 1 \Rightarrow -3 < k$

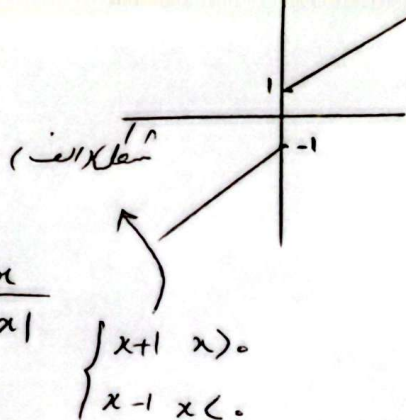
۳) $1 \leq x \rightarrow (3x - 3) - (x + 2) = k \Rightarrow 3x - 3 - x - 2 = k \Rightarrow 2x - 5 = k \Rightarrow x = \frac{k + 5}{2}$

$1 \leq \frac{k + 5}{2} \Rightarrow -3 \leq k$

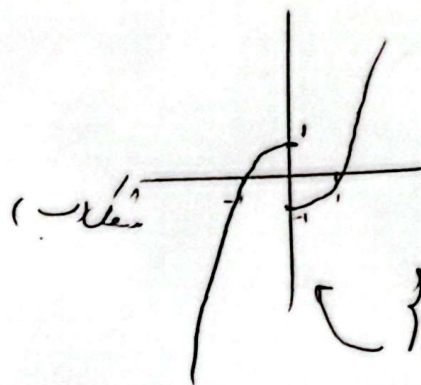
برای $k \in [-3, 9]$

$k \in [-3, 9]$

الف) $y = x + \frac{x}{|x|}$



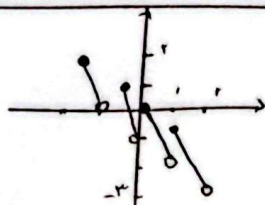
ب) $y = x/|x| - \frac{x}{|x|}$



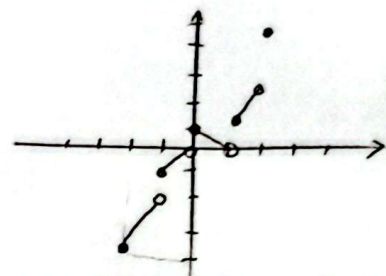
$\begin{cases} x+1 > 0 \\ x-1 < 0 \end{cases}$

$\begin{cases} x^2-1 > 0 \\ -x^2+1 < 0 \end{cases}$

الف) $y = [x] - rx$



ب) $y = [x]/|x|$



الف) $y = rx - [rx] \rightarrow Ry = [0, 1)$

ب) $y = rx - r[x] \rightarrow r(x - [x]) \Rightarrow Ry = [0, \varepsilon]$

ج) $y = x - \omega \left[\frac{x}{\omega} \right] \Rightarrow k \leq \frac{x}{\omega} < k+1 \Rightarrow \omega k \leq x < \omega(k+1) \Rightarrow 0 \leq x - \omega k < \omega \Rightarrow$

$\Rightarrow y = [rx] - rx \Rightarrow Ry = (-1, 0]$

$Ry = [0, \omega)$

الف) $y = r \sin x + r \cos x$

$a \sin x + b \cos x$

$-\sqrt{a^2+b^2} \leq r \sin x + r \cos x \leq \sqrt{a^2+b^2} \rightarrow Ry = [-\sqrt{r^2}, \sqrt{r^2}]$

ب) $y = r \sin x - r \cos x$

$a \sin x + b \cos x$

$a = r, b = -r \Rightarrow \sqrt{r^2 + (-r)^2} = \sqrt{2}r \Rightarrow Ry = [-\sqrt{2}r, \sqrt{2}r]$

ج) $y = [r \sin x + \omega \cos x] = -\sqrt{r^2} \leq r \sin x + \omega \cos x \leq \sqrt{r^2}$

$\Rightarrow y = \sqrt{\cos x - r \sin x}$

$Ry = \{ \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \}$

$-\sqrt{a} \leq \cos x - r \sin x \leq \sqrt{a} \Rightarrow Ry = [0, \sqrt{a}]$

⑤

$$\max(y) = y(1) = \gamma$$

$$Ry = [v_1, v]$$

$$Ry = [1, f]$$

$$\cot = \frac{\cos x}{\sin x} \rightarrow \frac{1}{\sin x} = \csc x \Rightarrow \frac{1}{\sin x} = \csc x$$

$$y = r \cot x \times \sin^2 x = r \frac{\cos x}{\sin x} \times \sin^2 x = \boxed{r \sin x \cos x = \sin^2 x} \quad y = \sin^2 x$$

$$Ry = [-1, 1]$$

$$\sin^2 x + \cos^2 x = 1$$

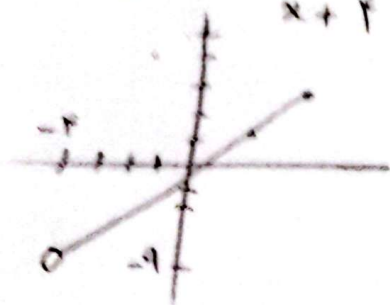
$$\bullet \text{ } \frac{1}{k} \leq \gamma_{\beta, \alpha}^{\gamma} \leq 1 \rightarrow \frac{1}{k} \gamma_{\beta, \alpha}^{\gamma} \in [0, \frac{1}{k}] \Rightarrow y = 1 - \frac{1}{k} \gamma_{\beta, \alpha}^{\gamma} \in [1 - \frac{1}{k}, 1] \Rightarrow$$

ب) $y = [\sin^{-1} x + \cos^{-1} x]$

$$r^{1-x} \leq \sin^x x + \cos^x x \leq 1 \quad \text{فقط} \quad [0,1] \rightarrow \text{مستقيم}$$

1.

الف) $y = \frac{1x^2 + 7x - 2}{x + 2} \Rightarrow \frac{(x+2)(1x-1)}{(x+2)} \quad D_f = \mathbb{R} - \{-2\}$

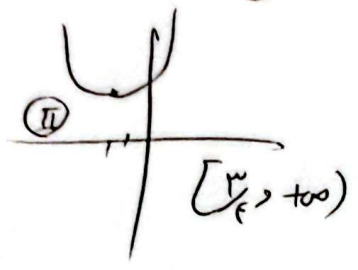


$E_f = \{R - \{-2\} - 1\}$

ب) $y = \frac{x^2 - 1}{x - 1} \Rightarrow \frac{(x-1)(x+1)}{(x-1)} \quad D_f = \mathbb{R} - \{1\}$
 $\textcircled{I} \mathbb{R} - \{1\} = \mathbb{R} \setminus \{1\}$

$x^2 + x + 1$

$x = \frac{-b}{2a} = -\frac{1}{2}, y = \frac{3}{4}$



$I \cap II \Rightarrow \left[-\frac{1}{2}, 1 \right) \cup (1, +\infty) \right]$
 مخرج