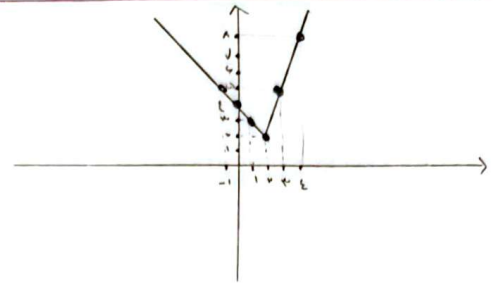


سوال ۱۵

دایره $y = x + |2x - 4|$

۲				
$x = 2x + 4$	$x + 2x - 4$			
$x + 4$	$3x - 4$			
$x \mid 2 \mid 0 \mid -1$	$x \mid 2 \mid 3 \mid 4$			
$y \mid 4 \mid 2 \mid 4 \mid 0$	$y \mid 2 \mid 0 \mid 1 \mid 1$			

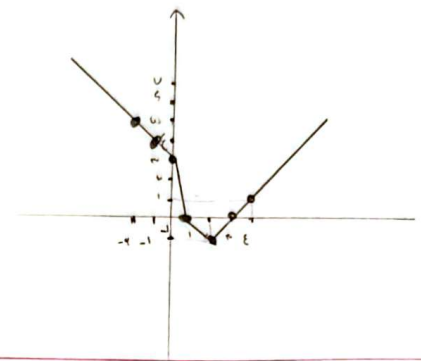


$R_f = [2, +\infty)$

۵

دایره $y = |x + 1| + |x - 1| - |x|$

۰				
$-x + 1$	$-3x + 1$	$-x + 1$	$x - 1$	
$x \mid 0 \mid -1 \mid -2$	$x \mid 0 \mid 1 \mid 0$	$x \mid 1 \mid 2 \mid 1$	$x \mid 2 \mid 3 \mid 4$	
$y \mid 1 \mid 0 \mid 1 \mid 2$	$y \mid 0 \mid 1 \mid 1 \mid 0$	$y \mid 0 \mid -1 \mid 1 \mid 1$	$y \mid -1 \mid 0 \mid 1 \mid 1$	



$R_f = [-1, +\infty)$

سوال ۱۶

$|3x - 3| - |x + 2| = k$

-۲				
$-3x + 3 + x + 2$	$-3x + 3 - x - 2$			
$-2x + 5$	$-4x + 1$			
$x \mid 0 \mid 1 \mid 2$	$x \mid 0 \mid 1 \mid 2$			
$y \mid 5 \mid 3 \mid 1$	$y \mid 1 \mid -3 \mid -5$			

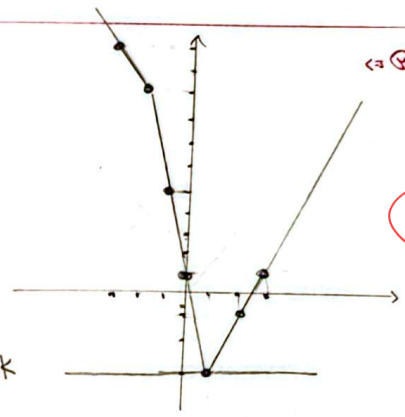
بسته تابع یک متغیری است. $\Rightarrow$ در $(-\infty, 0]$ تابع نزولی، در $[0, +\infty)$ تابع صعودی. $\min x = 1$ است.

$f(1) = |3(1) - 3| - |1 + 2| = |0| - |3| = 0 - 3 = -3$

$-3 = k$

آنگاه معادله فقط یک جواب داشته باشد پس تنها k ف توار -3 برابر -3 باشد.

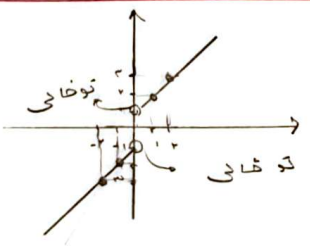
$k = -3$



۵

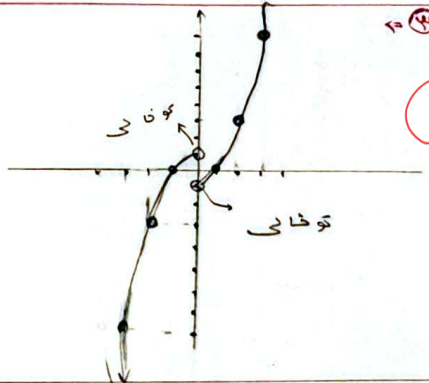
دایره $y = x + \frac{x}{|x|}$

۰				
$x = 1$	$x = -1$			
$x \mid 0 \mid -1 \mid 1$	$x \mid 1 \mid 2 \mid 3$			
$y \mid 0 \mid -1 \mid 1$	$y \mid 1 \mid 2 \mid 3$			



دایره $y = x|x| - \frac{x}{|x|}$

۰				
$-x^2 + 1$	$x^2 - 1$			
$x \mid -1 \mid 0 \mid 1$	$x \mid 1 \mid 2 \mid 3$			
$y \mid 0 \mid -1 \mid 0$	$y \mid 0 \mid 1 \mid 8$			

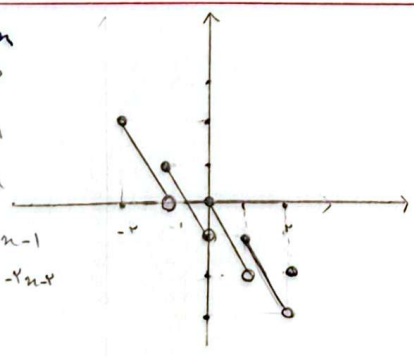


سوال ۱۷

۵

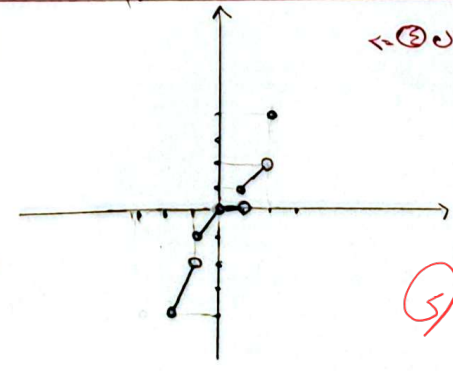
دایره $y = [x] - 2x$

- $x = 2 \rightarrow y = -4$
- $1 \leq x < 2 \rightarrow y = x + 1$
- $0 \leq x < 1 \rightarrow y = -2x$
- $-1 \leq x < 0 \rightarrow y = x - 1$
- $-2 \leq x < -1 \rightarrow y = x + 2$



دایره $y = [x]|x|$

- $1 \leq x < 2 \rightarrow y = x$
- $0 \leq x < 1 \rightarrow y = 0$
- $-1 \leq x < 0 \rightarrow y = -x$
- $-2 \leq x < -1 \rightarrow y = -2x$
- $x = 2 \rightarrow y = 2$



سوال ۱۸

۵

$$\text{مثال 1) } y = \sqrt{x} - \lfloor \sqrt{x} \rfloor \rightarrow R_f = [0, 1)$$

$$0 \leq \sqrt{x} - \lfloor \sqrt{x} \rfloor < 1$$

$$\hookrightarrow y = \sqrt{x} - \lfloor \sqrt{x} \rfloor \rightarrow \lfloor \sqrt{x} - \lfloor \sqrt{x} \rfloor \rfloor \rightarrow R_f = [0, 1)$$

$$2) y = \sqrt{x} - \lfloor \frac{x}{a} \rfloor \rightarrow \frac{x}{a} - \lfloor \frac{x}{a} \rfloor \rightarrow \lfloor \frac{x}{a} - \lfloor \frac{x}{a} \rfloor \rfloor \rightarrow R_f = [0, 1)$$

$$3) y = \lfloor \sqrt{x} \rfloor - \sqrt{x} \rightarrow R_f = [-1, 0]$$

$$\text{مثال 2) } y = \sqrt{x} \sin x + \sqrt{x} \cos x \rightarrow -\sqrt{x+1} \leq \sqrt{x} \sin x + \sqrt{x} \cos x \leq \sqrt{x+1} \rightarrow R_f = [-\sqrt{x+1}, \sqrt{x+1}]$$

$$\hookrightarrow y = \sqrt{x} \sin x - \sqrt{x} \cos x \rightarrow -\sqrt{(x+1)^2} \leq \sqrt{x} \sin x - \sqrt{x} \cos x \leq \sqrt{(x+1)^2} \rightarrow R_f = [-x-1, x+1]$$

$$2) y = [\sqrt{x} \sin x + \sqrt{x} \cos x] \rightarrow -\sqrt{x+1} \leq \sqrt{x} \sin x + \sqrt{x} \cos x \leq \sqrt{x+1} \rightarrow [-\sqrt{x+1}, \sqrt{x+1}] \rightarrow R_f = \{y \mid y \leq 2, -9 \leq y \leq 6\}$$

$$3) y = \sqrt{\cos x - \sin x} \rightarrow -\sqrt{a} \leq \cos x - \sin x \leq \sqrt{a} \rightarrow \sqrt{[-a, a]} \rightarrow R_f = [0, \sqrt{a}]$$

$$\text{مثال 3) } y = \sin^2 x + \sqrt{x} \sin x + \sqrt{x}$$

$$x=1 \rightarrow y=9$$

$$x=-1 \rightarrow y=0$$

$$\frac{-b}{2a} = \frac{-\sqrt{x}}{2} \rightarrow y = \frac{-1}{2} \approx 0.5$$

$$\Rightarrow R_f = [0, 9]$$

$$\hookrightarrow y = \sqrt{x} \sin^2 x + \sqrt{x} \sin x + \sqrt{x}$$

$$x=1 \rightarrow y=9$$

$$x=-1 \rightarrow y=1$$

$$\frac{-b}{2a} = \frac{-\sqrt{x}}{2} \rightarrow y = \frac{1}{2}$$

$$\Rightarrow R_f = [\frac{1}{2}, 9]$$

$$\text{مثال 4) } y = \sin^2 x + \sqrt{x} \cos^2 x \rightarrow \sin^2 x + \cos^2 x + \sqrt{x} \cos^2 x \rightarrow R_f = [1, 2]$$

$$\star -1 \leq \cos^2 x \leq 1$$

$$\hookrightarrow y = \frac{\sqrt{x} \cos^2 x}{1 + \cos^2 x} = \frac{\frac{\sqrt{x} \cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} = \frac{\sqrt{x} \cos^2 x \sin^2 x}{\sin^2 x} = \sqrt{x} \cos^2 x \sin^2 x = \sin^2 x \Rightarrow R_f = [-1, 1]$$

$$\text{مثال 5) } y = \sin^4 x + \cos^4 x \rightarrow \sqrt{1-x} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow \frac{1}{2} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow R_f = [\frac{1}{2}, 1]$$

$$\hookrightarrow y = [\sin^4 x + \cos^4 x] \rightarrow \sqrt{1-x} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow \frac{1}{2} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow [\sin^4 x + \cos^4 x] = \begin{cases} 0 \\ 1 \end{cases} \Rightarrow R_f = \{0, 1\}$$

$$\text{مثال 6) } y = \frac{x^2 + \sqrt{x} - 2}{x+2} \rightarrow \frac{(x+2)(\sqrt{x}-1)}{(x+2)} = \sqrt{x}-1 \quad ; \quad x \neq -2 \rightarrow y \neq -9 \Rightarrow R_f = \mathbb{R} - \{-9\}$$

$$\hookrightarrow y = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{(x-1)} = x+1 \quad ; \quad x \neq 1 \rightarrow y \neq 2$$

$$R_f = [\frac{3}{2}, +\infty)$$

$$\Rightarrow R_f = [\frac{3}{2}, +\infty)$$

