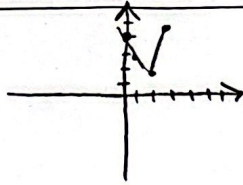


الف) $y = x + |x - 1|$

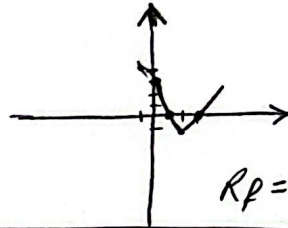
$$\begin{array}{c|c} x-1 & x+1 \\ \hline x-1 & x+1 \end{array}$$



$$R_f = [2, +\infty)$$

ب) $y = |x + 1| + |x - 1| - |x|$

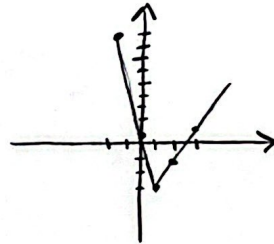
$$\begin{array}{c|c|c|c} x+1 & x-1 & -x & \\ \hline x+1 & x-1 & -x & \end{array}$$



$$R_f = [-1, +\infty)$$

ج) $|x - 3| - |x + 2| = k$

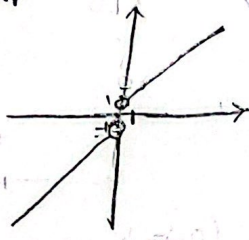
$$\begin{array}{c|c|c} x-3 & -x-2 & \\ \hline x-3 & -x-2 & \end{array}$$



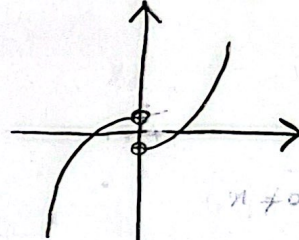
برای اینکه به رابطه داشته باشد
 $k = -3$

الف) $y = x + \frac{x}{|x|}$

$$\begin{array}{l} x > 0 \quad x + 1 \\ x < 0 \quad x - 1 \\ 0 \quad \text{تنگ} \end{array}$$

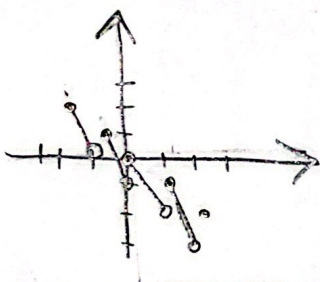


ب) $y = x|x| - \frac{x}{|x|}$

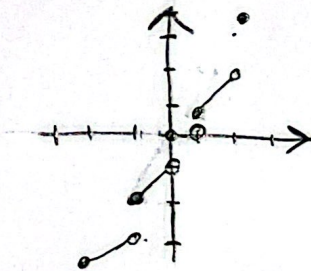


$$\begin{array}{l} x > 0 \quad x^2 - 1 \\ x < 0 \quad -x^2 + 1 \\ 0 \quad \text{تنگ} \end{array}$$

الف) $y = [x] - 2x$



ب) $y = [x]|x|$



الف) $y = x - [x]$

$$0 \leq x - [x] < 1$$

$$[0, 1)$$

ب) $x - \frac{x}{[x]}$

$$x - \frac{x}{[x]} \in [0, 1)$$

$$R_f = [0, \infty)$$

ب) $y = x - [x]$

$$y = x - [x] \in [0, 1)$$

$$\begin{array}{l} y = x - [x] \\ -1 < [x] - x \leq 0 \\ (-1, 0] \end{array}$$

$$-\sqrt{a^2+b^2} \leq a \sin \alpha + b \cos \alpha \leq \sqrt{a^2+b^2}$$

$$\text{a) } y = r \sin \alpha + r \cos \alpha$$

$$-\sqrt{9+4} \leq r \sin \alpha + r \cos \alpha \leq \sqrt{9+4}$$

$$-\sqrt{13} \leq r \sin \alpha + r \cos \alpha \leq \sqrt{13}$$

$$[-\sqrt{13}, \sqrt{13}]$$

$$\Rightarrow y = r \sin \alpha - r \cos \alpha$$

$$-\sqrt{14+9} \leq r \sin \alpha - r \cos \alpha \leq \sqrt{14+9}$$

$$-\sqrt{23} \leq r \sin \alpha - r \cos \alpha \leq \sqrt{23}$$

$$[-\sqrt{23}, \sqrt{23}]$$

$$\text{2) } y = [r \sin \alpha + a \cos \alpha]$$

$$-\sqrt{19} \leq r \sin \alpha + a \cos \alpha \leq \sqrt{19}$$

$$-9 \leq [r \sin \alpha + a \cos \alpha] \leq a$$

$$\Rightarrow y = \sqrt{\cos \alpha - r \sin \alpha}$$

$$-\sqrt{4+1} \leq \cos \alpha - r \sin \alpha \leq \sqrt{4+1}$$

$$-\sqrt{5} \leq \cos \alpha - r \sin \alpha \leq \sqrt{5}$$

$$[0, \sqrt{5}]$$

$$R_f = \{-9, -a, -f, -r, -1, 0, 1, r, a, f\}$$

$$\sqrt{0} \leq \sqrt{\cos \alpha - r \sin \alpha} \leq \sqrt{5}$$

$$\text{a) } y = \sin^2 \alpha + r \sin \alpha + r$$

$$\sin \alpha \begin{matrix} \xrightarrow{1} \\ \xrightarrow{-\frac{b}{a} = -\frac{r}{r} = -1} \end{matrix}$$

$$R_f = [0, 4]$$

$$\Rightarrow y = r \sin^2 \alpha + r \sin \alpha + r$$

$$\sin \alpha \begin{matrix} \xrightarrow{1} \\ \xrightarrow{-\frac{b}{a} = -\frac{r}{r} = -1} \end{matrix}$$

$$R_f = [\frac{11}{14}, \frac{12}{14}]$$

$$\text{a) } y = \sin^2 \alpha + r \cos^2 \alpha$$

$$y = \sin^2 \alpha + r(1 - \sin^2 \alpha) = \sin^2 \alpha + r - r \sin^2 \alpha = -r \sin^2 \alpha + r$$

$$R_f = [1, r]$$

$$\Rightarrow y = \frac{r \cos^2 \alpha}{1 + r \sin^2 \alpha}$$

$$\frac{\frac{r \cos^2 \alpha}{\sin^2 \alpha}}{\frac{1}{\sin^2 \alpha}} = r \cos^2 \alpha = \sin^2 \alpha$$

$$R_f = [1, 1]$$

$$\text{a) } y = \sin^4 \alpha + \cos^4 \alpha$$

$$rK=4 \quad K=r$$

$$r^{-r} \leq \sin^4 \alpha + \cos^4 \alpha \leq 1$$

$$\frac{1}{r} \leq \sin^4 \alpha + \cos^4 \alpha \leq 1$$

$$[\frac{1}{r}, 1]$$

$$\Rightarrow y = [\sin^4 \alpha + \cos^4 \alpha]$$

$$rK=1 \quad K=r$$

$$\frac{1}{\lambda} \leq \sin^4 \alpha + \cos^4 \alpha \leq 1$$

$$r^{-r} \leq \sin^4 \alpha + \cos^4 \alpha \leq 1$$

$$[0, 1]$$

$$\text{a) } y = \frac{r n^r + u n - r}{n + r}$$

$$\frac{\frac{r n^r + u n - r}{n + r}}{\frac{r n^r + u n}{n + r}} = \frac{n + r}{r n - 1}$$

$$y = \frac{(r n - 1)(n + r)}{(n + r)} = r n - 1$$

$$R_f = [r - 1, r]$$

$$\Rightarrow y = \frac{n^r - 1}{n - 1} = \frac{(n - 1)(n^{r-1} + n^{r-2} + \dots + 1)}{(n - 1)} = n^{r-1} + n^{r-2} + \dots + 1$$

$$\frac{-b}{r a} = -\frac{1}{r}$$

$$R_f = [\frac{r}{r}, \infty)$$

$$\frac{1}{r} - \frac{r}{r} + \frac{r}{r} = \frac{r}{r} \rightarrow y_{min}$$

