

الف) $y = x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow R_y = \mathbb{R}$ (سوال ۱)

ب) $y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow \Delta = 4y^2 - 4y(-1) = 4y^2 + 4y$

$\omega_p \rightarrow \frac{2 \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y^2 + 4y \geq 0 \rightarrow y(y+1) \geq 0$
 $y \neq 0$
 $y \neq -1$

الف) $y = x^2 - 4x + 10 \rightarrow x_s = \frac{-(-4)}{2} = 2 \rightarrow y_s = 4 - 8 + 10 = 6$ (سوال ۲)
 $R_y = [6, +\infty)$

ب) $y = -x^2 + 4x + 3 \rightarrow x_s = 2 \rightarrow y_s = -4 + 8 + 3 = 7 \Rightarrow (-\infty, 7]$

ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow \Delta = 16 - 4(1)(-3) = 28 \rightarrow \frac{-\Delta}{2a} = \frac{-28}{2} = -14$

د) $y = \sqrt{4x - x^2} \xrightarrow{a < 0} x_{\max} = \frac{-b}{2a} = \frac{2}{-1} = -2 \rightarrow y_{\max} = 2 \rightarrow R_y = (-\infty, 2]$
 $y_s = [-1, +\infty)$

الف) $R_y = \mathbb{R}$ ب) $R_y = \mathbb{R}$ ج) $R_y = [0, +\infty)$ (سوال ۳)

الف) $y = \frac{3x+1}{x-2} = \mathbb{R} - \{3\} = R_y$ (سوال ۴)

ب) $y = \frac{2x+1}{x+1} \Rightarrow R_y = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{2x+4}{x+1}} \rightarrow y^2 x + y^2 = 2x+4 \rightarrow y^2 x - 2x = 4 - y^2$ (سوال ٧)

$$x = \frac{y^2 + 4}{y^2 - 2}$$

$$x(y^2 - 2) = y^2 + 4$$

$$y^2 - 2 \rightarrow y = \sqrt{2} \rightarrow \sqrt{2} \rightarrow x = \frac{-\sqrt{2} + \sqrt{2}}{+1 - 1} +$$

$$R_y = \mathbb{R} - [-\sqrt{2}, \sqrt{2}]$$

ب) $y = \sqrt{\frac{4x+1}{2-x}} \rightarrow 2y^2 - y^2 x = 4x+1 \rightarrow 2y^2 - 1 = 4x + y^2 x$

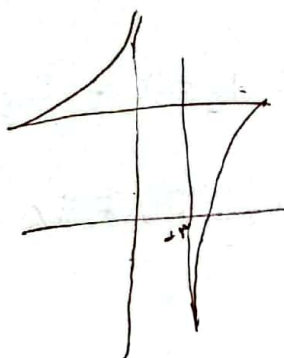
$$2y^2 - 1 = x(4 + y^2) \rightarrow x = \frac{2y^2 - 1}{4 + y^2} \rightarrow$$

$$x = \frac{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{+1 - 1} +$$

$$R_y = \mathbb{R} - \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$$

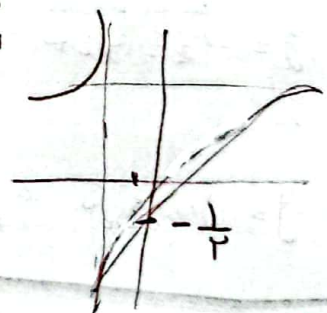
الف) $y = \frac{2x^3}{x+1}$

بجانب قائم $\rightarrow -1$
بجانب افقی $\rightarrow 2$



ب) $y = \frac{2x-1}{x+2}$

بجانب قائم: $x = -2$
بجانب افقی: $y = 2$



الف) $y = \sin x + \frac{1}{\sin x} \rightarrow R_y = (-\infty, -2] \cup [2, +\infty)$ (سوال ٧)

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3} \rightarrow R_y = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt{2x^2+1}}{y x} \rightarrow \sqrt{2x^2+1} = y^2 x \rightarrow R_y = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_y = [2, +\infty)$

$$a) y = x^2 + \frac{1}{x^2 + 2x} \rightarrow x^2 + \frac{1}{x^2 + 2x} + \frac{1}{x} \rightarrow R_y = \left[\frac{1}{x}, \infty \right) \quad (\text{سوال 1})$$

$$b) y = \frac{x^2 + k + 1}{\sqrt{x^2 + k}} = \sqrt{x^2 + k} + \frac{1}{\sqrt{x^2 + k}} \rightarrow$$

$$\sqrt{k} + \frac{1}{\sqrt{k}} \sin = -k \min$$

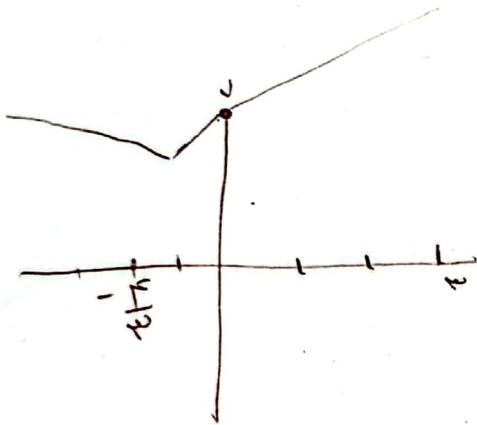
$$\frac{\sqrt{k} + 1}{\sqrt{k}} = \frac{a}{\sqrt{k}} \times \frac{\sqrt{k}}{\sqrt{k}} = \frac{a \sqrt{k}}{k} = \frac{a x^2}{x^2 k} = \frac{1}{k}$$

$$R_y = [1, \infty)$$

$$y = |x - 3| + |3x + k| \rightarrow$$

$$\begin{cases} x - 3 + 3x + k \\ 3 - x + 3x + k \\ -x + 3 - 3x - k \\ -kx - 3 \end{cases}$$

$$\begin{aligned} x &> 3 \quad (\text{سوال 9}) \\ \frac{k}{3} &\leq x \leq 3 \\ x &< \frac{k}{3} \end{aligned}$$



$$a) y = |x - 2| + |2x + 2|$$

$$b) y = |2x + 2| - |x + 1| \quad (\text{سوال 10})$$

$$\begin{matrix} |1| & |2| & |3| \\ |0| & |1| & |2| \end{matrix}$$

$$\begin{matrix} |1| & |2| & |3| \\ |0| & |1| & |2| \end{matrix}$$



$$R_y = [3, \infty)$$

$$\begin{matrix} |1| & |2| & |3| \\ |0| & |1| & |2| \end{matrix}$$

